2629

273

AD 696472

TECHNICAL REPORT NO. 10561

AD696472

FAMSNUB, A COMPUTER PROGRAM FOR DETERMINING THE FREQUENCIES AND MODE SHAPES OF NONUNIFORM BEAMS

U.S. ARMY TANK-AUTOMOTIVE COMMAND LABORATORIES

MECHANICAL BRANCH
AUTOMOTIVE COMPONENTS DIVISION

September 1969

by John L. McFarland

TACOM

VEHICULAR COMPONENTS & MATERIALS LABORATORY

U.S. ARMY TANK AUTOMOTIVE COMMAND Warren, Michigan

AD27152

20020808058

The findings in this report are not to be construed as an official Department of the Army position.

Citation of equipment in this report does not constitute an official indorsement or approval of the use of such commercial hardware.

The citation of commercial products in this report does not constitute an official indorsement or approval of such products.

Distribution of this document is unlimited.

DESTROY THIS REPORT WHEN
IT IS NO LONGER NEEDED.
DO NOT RETURN IT.

TECHNICAL REPORT NO. 10561

FAMSNUB, A COMPUTER PROGRAM FOR DETERMINING THE FREQUENCIES AND MODE SHAPES OF NONUNIFORM BEAMS

J.L. McFarland

September 1969

Distribution of this document is unlimited

DA Project No: 1T061101A91A
Task Title: In-House Initiated Lab R&D
Task Area: 00
Work Unit No: 205EH

Mechanical Branch
Automotive Components Division
Vehicular Components and Materials Laboratory
US Army Tank-Automotive Command

TABLE OF CONTENTS

ABSTRACT

FAMSNUB is a digital computer program, written in single precision FORTRAN IV for the Philco-Ford/GE 265 time-sharing system, to determine the eigenfrequencies and eigenmodes of vibration in pure bending of loaded, piece-wise continuous single-span beams supported in any one of the ten possible combinations of the following end conditions: free, guided, hinged and clamped. Continuously variable beam properties may be accommodated by appropriate subdivision of the span. The distributed mass of the beam itself is taken into account by means of field transfer matrices; this represents an advance over the usual lumped-parameter approach. To preserve numerical accuracy at the higher mode values, a modified transfer matrix method is employed. Verification solutions have been obtained on the IBM 360-50/65 using the double precision facility.

ILLUSTRATIONS AND TABLES

FOREWORD

This report documents results obtained from project work on task title "In-House Initiated Lab R & D" (DA Proj. No. 1T061101A91A/Task Area 00/Work Unit No. 205EH) authorized by Dr. E.N. Petrick, USATACOM Chief Scientist/Technical Director of Laboratories. The project is intended to strengthen in-house research capabilities and provide a methodology for dealing with practical problems. The author extends his sincerest appreciation to Dr. Petrick for the opportunity to perform this work and to his senior colleague, Mr. Douglas J. Hackenbruch for the encouragement and assistance he has provided during all phases of this project.

INTRODUCTION

This work was initiated after a literature search revealed that no generally applicable computerized solution for beam vibration eigenvalues had yet been developed which took account of the continuity of beam properties. The analytical development contained herein has been applied to piece-wise continuous beams taking account of bending only. Extensions which would include continuous variation of properties, the effects of shear deflection and rotary inertia, intermediate support conditions and response to forcing functions are quite feasible.

Conventional numerical methods of determining natural frequencies of transverse beam vibration generally employ variations of lumped-parameter techniques. To account in a more refined way for the distributed mass of the elastic system, field transfer matrices, which will be developed in the following section, have been incorporated into FAMSNUB and combined with point transfer matrices to accommodate discrete loading. In addition to discrete loading, step-wise variations can be accommodated in: uniformly-distributed loading, beam material density, elastic modulus and section modulus.

The source language employed should provide greater time-sharing system compatibility insofar as no use is made of subscripted subscripts, mixed modes, variable name lengths exceeding six characters nor of the double precision facility. To accommodate users of time-sharing systems, program length limitations have been observed by subdividing FAMSNUB into 5 saved items dealing with the conditioning of input data, eigenfrequency calculation (3 parts) and the computation of normal modes. This has been accomplished through the employment of intermediate permanent file storage and the $USE monitor line.

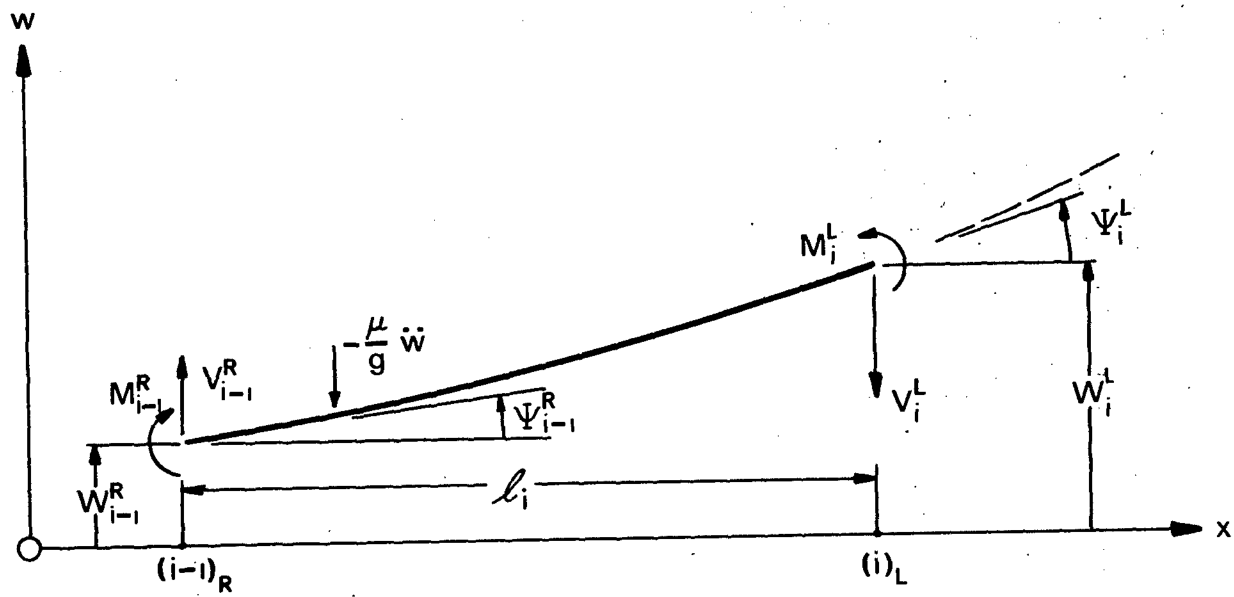
The considerable numerical magnitude difficulties encountered in evaluating 2x2 frequency determinants, which involve taking the difference between quantities which are several orders of magnitude larger than the difference itself, are sidestepped by an adaptation of a method developed by Pestel & Leckie (1), pp. 204-213.

FAMSNUB commences with the first non-zero frequency and proceeds continuously to the mode(s) of interest. However, a start may occur at any frequency to investigate a resonance condition. If the eigenmodes are also desired, a printout can be obtained in four-column format of deflection amplitude, maximum bending stress and average transverse shear stress, together with the corresponding distance along the beam (Figure 27).

OBJECT

Develop a working method for computing the eigenvalues and eigenmodes of transverse beam vibration, taking account of the distributed beam mass in preference to employing a lumped-parameter approach.

PROBLEM DESCRIPTION



Forces and Deflections for Beam Element

Figure 1

Field and Point Transfer Matrices

Consider an initially straight, piece-wise continuous beam performing free transverse vibrations. Figure 1 shows an isolated beam element of length ℓ_i between points $(i-1)$ and (i), having continuous elastic properties and a constant rate of inertia loading $-\frac{\mu}{g}\ddot{w}$. μ is the non-reactive loading per unit length due to beam self-weight and continuous loading. The origin of the coordinate plane, in which all vibration is assumed to occur,

is to be considered as coincident with the left-hand support. The sign convention is also revealed by Figure 1, which shows positive, instantaneous values of the state vector components: deflection w, slope Ψ (in radians), bending moment M and shear force V. The uniform bending stiffness for this length is $E_i I_i$, the product of the elastic modulus and the second moment of area of the cross-section.

From simple bending theory, the fourth partial derivative of $w(x,t)$ with respect to x is proportional to the rate of loading, thus

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} = -\frac{\mu}{g} \ddot{w} \tag{1}$$

For a solution which permits eigenfrequency determination we write w as the product of two single-variable functions

$$w(x,t) = W(x) \sin \omega t \tag{2}$$

where $W(x)$ is the dynamic amplitude and ω is the simple harmonic circular frequency; substituting into (1)

$$EI \frac{d^4 W}{dx^4} = \frac{\mu}{g} W \omega^2 \tag{3}$$

which becomes

$$\frac{d^4 W}{dx^4} - m^4 W = 0 \tag{4}$$

where

$$m = \sqrt[4]{\frac{\mu \omega^2}{gEI}} \tag{5}$$

Rewriting (4) in LaPlace transform notation

$$p^4 W(p) - p^3 W_o - p^2 W_o' - pW_o'' - W_o''' - m^4 W(p) = 0 \tag{6}$$

where $\quad W(p) = \int_o^\infty e^{-px} f(x)\,dx \qquad W_o^n = \left.\frac{d^n W}{dx^n}\right|_{x=0}$

solving $\quad W(p) = \frac{1}{p^4 - m^4}\left[p^3 W_o + p^2 W_o' + pW_o'' + W_o''' \right] \qquad (7)$

where $\quad W_o = W_{i-1}^R\,;\ W_o' = \Psi_{i-1}^R;\ E_i I_i W_o'' = M_{i-1}^R;\ E_i I_i W_o''' = V_{i-1}^R$

Making the appropriate substitutions and applying inverse transforms:

$$W(x) = W_i^L = \frac{\cos mx + \cosh mx}{2} W_{i-1}^R + \frac{\sin mx + \sinh mx}{2m} \Psi_{i-1}^R$$
$$+ \frac{\cosh mx - \cos mx}{2E_i I_i m^2} M_{i-1}^R + \frac{\sinh mx - \sin mx}{2E_i I_i m^3} V_{i-1}^R \tag{8}$$

$$W'(x) = \Psi_i^L = \frac{m(\sinh mx - \sin mx)}{2} W_{i-1}^R + \frac{\cos mx + \cosh mx}{2} \Psi_{i-1}^R$$
$$+ \frac{\sin mx + \sinh mx}{2E_i I_i m} M_{i-1}^R + \frac{\cosh mx - \cos mx}{2E_i I_i m^2} V_{i-1}^R \tag{9}$$

$$W''(x) = M_i^L = \frac{E_i I_i m^2(\cosh mx - \cos mx)}{2} W_{i-1}^R + \frac{E_i I_i m(\sinh mx - \sin mx)}{2} \Psi_{i-1}^R$$
$$+ \frac{\cos mx + \cosh mx}{2} M_{i-1}^R + \frac{\sin mx + \sinh mx}{2m} V_{i-1}^R \tag{10}$$

$$Z_i^L = F_i \cdot Z_{i-1}^R$$

$$\begin{bmatrix} W \\ \Psi \\ M \\ V \end{bmatrix}_i^L = \begin{bmatrix} \frac{\cos mx + \cosh mx}{2} & \frac{\sin mx + \sinh mx}{2m} & \frac{\cosh mx - \cos mx}{2EIm^2} & \frac{\sinh mx - \sin mx}{2EIm^3} \\ \frac{m(\sinh mx - \sin mx)}{2} & \frac{\cos mx + \cosh mx}{2} & \frac{\sin mx + \sinh mx}{2EIm} & \frac{\cosh mx - \cos mx}{2EIm^2} \\ \frac{EIm^2(\cosh mx - \cos mx)}{2} & \frac{EIm(\sinh mx - \sin mx)}{2} & \frac{\cos mx + \cosh mx}{2} & \frac{\sin mx + \sinh mx}{2m} \\ \frac{EIm^3(\sin mx + \sinh mx)}{2} & \frac{EIm^2(\cosh mx - \cos mx)}{2} & \frac{m(\sinh mx - \sin mx)}{2} & \frac{\cos mx + \cosh mx}{2} \end{bmatrix}_i \cdot \begin{bmatrix} W \\ \Psi \\ M \\ V \end{bmatrix}_{i-1}^R$$

Transfer Matrix for an Elastic Field

Figure 2

$$W'''(x) = V_i^L = \frac{E_i I_i m^3(\sin mx + \sinh mx)}{2} W_{i-1}^R + \frac{E_i I_i m^2(\cosh mx - \cos mx)}{2} \Psi_{i-1}^R$$

$$+ \frac{m(\sinh mx - \sin mx)}{2} M_{i-1}^R + \frac{\cos mx + \cosh mx}{2} V_{i-1}^R \qquad (11)$$

Hence the relation of the state vectors Z_{i-1}^R and Z_i^L on either side of an elastic field of length x can be stated as

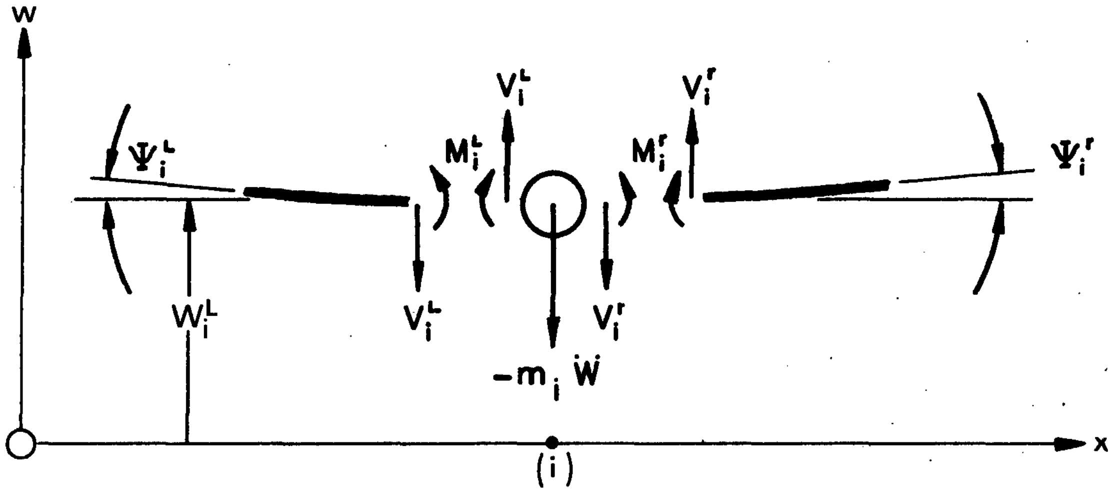
$$Z_i^L = F_i \cdot Z_{i-1}^R \qquad (12)$$

Equation (12) can be represented in matrix notation as shown in Figure 2. F_i is called the field transfer matrix which, by taking the limit as m approaches zero, reduces to the well-known transfer matrix for an elastic, massless field as shown in Figure 3.

$$\begin{bmatrix} W \\ \Psi \\ M \\ V \end{bmatrix}_i^L = \begin{bmatrix} 1 & x & \frac{x^2}{2EI} & \frac{x^3}{6EI} \\ 0 & 1 & \frac{x}{EI} & \frac{x^2}{2EI} \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1 \end{bmatrix}_i \cdot \begin{bmatrix} W \\ \Psi \\ M \\ V \end{bmatrix}_{i-1}^R$$

Transfer Matrix for a
Massless, Elastic Field

Figure 3



Free Body Diagram for the Point Mass m_i

Figure 4

$$Z_i^R = P_i \cdot Z_i^L$$

$$\begin{bmatrix} W \\ \Psi \\ M \\ V \end{bmatrix}_i^R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ m\omega^2 & 0 & 0 & 1 \end{bmatrix}_i \cdot \begin{bmatrix} W \\ \Psi \\ M \\ V \end{bmatrix}_i^L$$

Transfer Matrix for a Point Mass

Figure 5

With reference to the free body diagram for the point mass m_i situated at point (i) as shown in Figure 4, the state vector components on either side of m_i are related as follows:

$$W_i^R = W_i^L$$

$$\Psi_i^R = \Psi_i^L$$

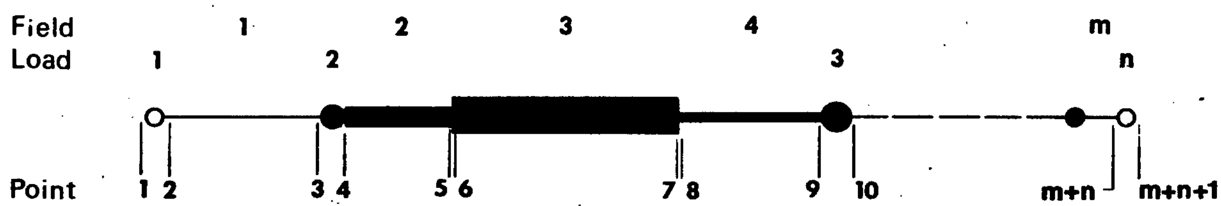
$$M_i^R = M_i^L$$

$$V_i^R = V_i^L + m_i W_i \omega^2 \qquad (13)$$

where ω is the harmonic circular frequency. Thus a relationship similar to (12) may be written

$$Z_i^R = P_i Z_i^L \qquad (14)$$

where P_i is the point transfer matrix. Equation (14) may be written in matrix notation as shown in Figure 5.



General, Piece-wise Continuous Beam

Figure 6

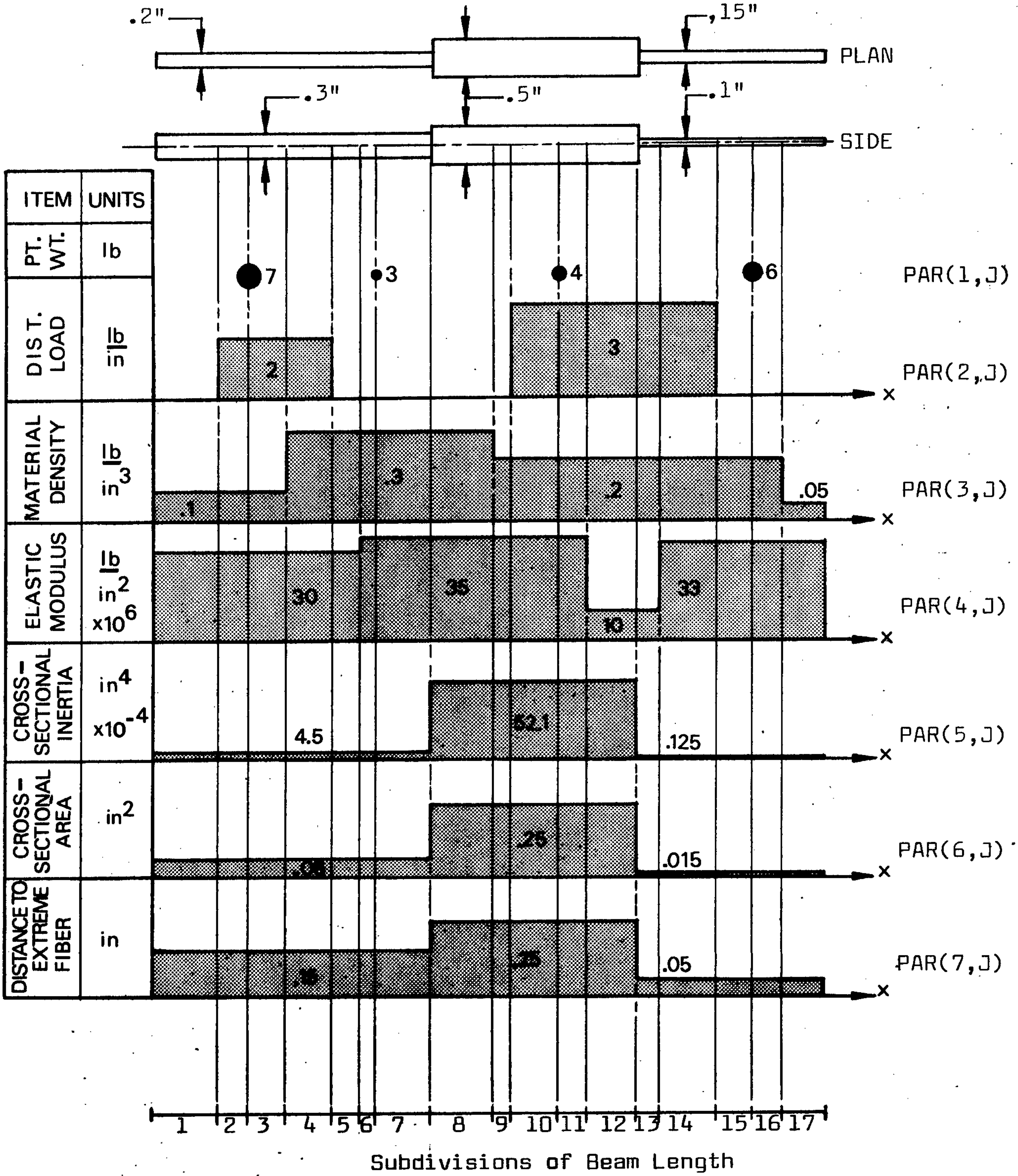
The relation between the state vectors Z_{m+n+1} and Z_1 at each end of the generalized beam shown in Figure 6, having **m** uniform elastic fields and **n** point loads will be of the form

$$\begin{aligned} Z_{m+n+1} &= P_1 \cdot F_1 \cdot P_2 \cdot F_2 \cdot F_3 \cdot F_4 \cdot \;\cdot\; \cdot F_m \cdot P_n \cdot Z_1 \\ &= U \cdot Z_1 \end{aligned} \qquad (15)$$

where the intermediate state vectors will have been eliminated by multiplication of the transfer matrices P_i and F_i to form the overall transfer matrix U. For the idealized end conditions treated in this report two columns of U can be ignored, owing to the two zero components of Z_1; also two components of Z_{m+n+1} will be zero, allowing the formation of a 2x2 determinant D.

A simple beam may be treated algebraically, in which case the coefficients of D would involve functions of ω. Solutions for ω_n would be obtained from the eigenfrequency condition $D=0$. Realistic problems demand a numerical approach in which ω_n is surmised and the value of D obtained. If the latter is plotted against ω, the eigenvalues can be bracketed with reasonable accuracy. However, using only eight digits may give rise to precision problems at low frequencies even for simple beams. A more generally applicable numerical approach has been adapted from Pestel and Leckie, (ref. 1, pp. 204-213), referred to therein as the "modified transfer matrix method". This method avoids evaluating U and hence D by directly obtaining the two (ideally) zero components of Z_{m+n+1}. One of the two nonzero components of Z_1 is normalized to unity, the other being arbitrarily chosen. For a given estimate of eigenfrequency, a single-valued frequency function $R(\omega)$ can be obtained after two iterations. The first iteration establishes the correction to be applied to the non-normalized component of Z_1 as the arithmetic mean of the two ideally zero components of Z_{m+n+1}. These latter components are again obtained and, at the eigenfrequency condition, will be zero; generally, however, their difference, which is termed $R(\omega)$, will be nonzero and can be considered analogous to the value of D at a general ω. This method has the advantage that numerical difficulties arising from evaluating D, where the difference is several orders of magnitude smaller than the minuend and subtrahend, have been eliminated. This process was the principal error source, and, using the modified method, high eigenfrequencies of complex beams can be determined with reasonable accuracy without recourse to the double precision facility.

(This page left intentionally blank)



.2"
,15"
PLAN
.3"
.5"
.1"
SIDE
ITEM
UNITS
PT. WT.
lb
7
3
4
6
PAR(1,J)
DIST. LOAD
lb/in
2
3
PAR(2,J)
MATERIAL DENSITY
lb/in3
.1
.3
.2
.05
PAR(3,J)
ELASTIC MODULUS
lb/in2 x10^6
30
35
10
33
PAR(4,J)
CROSS-SECTIONAL INERTIA
in4 x10^-4
4.5
52.1
.125
PAR(5,J)
CROSS-SECTIONAL AREA
in2
.06
.25
.015
PAR(6,J)
DISTANCE TO EXTREME FIBER
in
.15
.25
.05
PAR(7,J)
x
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
Subdivisions of Beam Length

FIGURE 7

Conditioning of Input Data

In its time-sharing form, as mentioned in the Introduction, FAMSNUB appears in 5 separate parts or saved items: EMB10 which conditions the input data; EMB15 which in conjunction with EMB20 and EMB21 computes the eigenfrequencies and state vectors; and EMB25 which computes the eigenmodes and stress values. The following description of FAMSNUB is made with reference to these time-sharing component items, which will in turn be illustrated by a sample problem. This problem very nearly exhausted the core storage capacity of the Philco-Ford system at the time of writing.

The beam dealt with is shown in Figure 7 opposite. Seven parameters PAR(I,J),I=1,7 are considered to describe fully the discrete and distributed loading systems and the beam's material and geometrical properties as follows:

PAR(1,J),J=1,4 are point weights of 7,3,4 and 6 pounds positioned 5.1,11.9,21.8 and 32.1 inches respectively from the left-hand support.

PAR(2,J),J=1,2 are uniformly-distributed loadings of 2 and 3 lb/in., applied over the lengths shown.

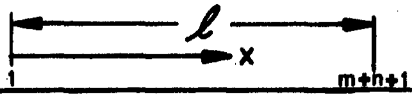
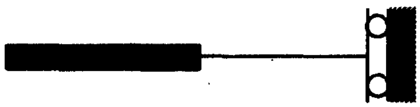
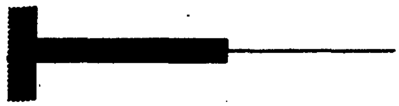
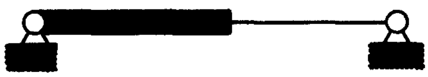
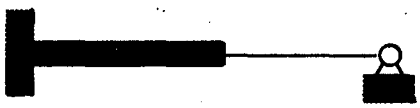
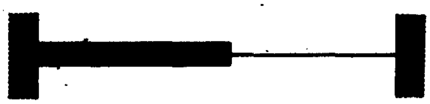
PAR(3,J),J=1,4 are the values of material density.

PAR(4,J),J=1,4 are the values of the elastic modulus.

PAR(5,J),J=1,3 are the values of area moment about the neutral axis.

PAR(6,J),J=1,3 are the values of cross-sectional area.

PAR(7,J),J=1,3 are the distances from the neutral axes of the various sections to the outermost bending fiber.

NSUPP		SUPPORT CONDITIONS	STATE VECTORS Z_1	STATE VECTORS Z_{m+n+1}
1		FREE-FREE	$\begin{matrix}1\\\lambda\\0\\0\end{matrix}$	$\begin{matrix}-\\-\\0\\0\end{matrix}$
2		FREE-GUIDED	$\begin{matrix}1\\\lambda\\0\\0\end{matrix}$	$\begin{matrix}-\\0\\-\\0\end{matrix}$
3		FREE-HINGED	$\begin{matrix}1\\\lambda\\0\\0\end{matrix}$	$\begin{matrix}0\\-\\0\\-\end{matrix}$
4		GUIDED-GUIDED	$\begin{matrix}1\\0\\\lambda\\0\end{matrix}$	$\begin{matrix}-\\0\\-\\0\end{matrix}$
5		GUIDED-HINGED	$\begin{matrix}1\\0\\\lambda\\0\end{matrix}$	$\begin{matrix}0\\-\\0\\-\end{matrix}$
6		CLAMPED-FREE	$\begin{matrix}0\\0\\\lambda\\1\end{matrix}$	$\begin{matrix}-\\-\\0\\0\end{matrix}$
7		HINGED-HINGED	$\begin{matrix}0\\\lambda\\0\\1\end{matrix}$	$\begin{matrix}0\\-\\0\\-\end{matrix}$
8		CLAMPED-HINGED	$\begin{matrix}0\\0\\\lambda\\1\end{matrix}$	$\begin{matrix}0\\-\\0\\-\end{matrix}$
9		CLAMPED-GUIDED	$\begin{matrix}0\\0\\\lambda\\1\end{matrix}$	$\begin{matrix}-\\0\\-\\0\end{matrix}$
10		CLAMPED-CLAMPED	$\begin{matrix}0\\0\\\lambda\\1\end{matrix}$	$\begin{matrix}0\\0\\-\\-\end{matrix}$

End Constraints and State Vectors

Figure 8

```
EMB10R
         NSS   NOSUP   NMINT   NMSHP   NOINTP      G        BMLGTH         NPAR(I)
100      17      3       5       5       10       386.       36.    {4   5   4   4   3   3   3}
105          (3.4             1.7            2.1         2.4          1.4            .9
110    SS(I){3.0             3.4             .8         2.7          2.1           2.1
115          (1.1             3.1            1.9         1.7          2.2
120  NSUPP(I) {2   7   10}  {1   2   3   4   5}  {1   2   3   4   5}
125      .7000E 01   3 )            MINT(I)             MSHP(I)
130      .3000E 01   7 } PAR(1,J)
135      .4000E 01  11 }
140      .6000E 01  16 )
145      .0000E 00   1 )
150      .2000E 01   2 }
155      .0000E 00   5 } PAR(2,J)
160      .3000E 01  10 }
165      .0000E 00  15 )
170      .1000E 00   1 )
175      .3000E 00   4 } PAR(3,J)
180      .2000E 00   9 }
185      .5000E-01  17 )
190      .3000E 08   1 )
195      .3500E 08   6 } PAR(4,J)
200      .1000E 08  12 }
205      .3300E 08  14 )
210      .4500E-03   1 )
215      .5210E-02   8 } PAR(5,J)
220      .1250E-04  13 )
225      .6000E-01   1 )
230      .2500E 00   8 } PAR(6,J)
235      .1500E-01  13 )
240      .1500E 00   1 )
245      .2500E 00   8 } PAR(7,J)
250      .5000E-01  13 )
```

Initial Data Set EMB1OR

Figure 9

The NOSUP=10 different support conditions are illustrated in Figure 8, each condition corresponding to a value of the subscripted variable NSUPP. The state vectors Z and Z_{m+n+1} are given in forms corresponding to the requirements of the modified transfer matrix method, i.e. the two non-zero components of Z_1 appear as unity and λ, which represents the initial, arbitrary choice of slope or bending moment as the case may be. For the sample problem, three support conditions are chosen: NSUPP(1)=2 (free-guided), NSUPP(2)=7 (hinged-hinged) and NSUPP(3)=10 (clamped-clamped). The order of support condition can in general be accounted for by taking the origin of x at the appropriate end; thus if for the sample problem the initial support condition had been guided-free, the right-hand end would have been taken as the origin. It would be feasible for all ten conditions to be included in one run on a batch system, but for time-sharing purposes the saved items EMB15/EMB20/EMB21 will terminate if the eigenmodes and stress values are desired, and EMB25 will have to be run separately. The input data are contained in file EMB10R as shown in Figure 9. EMB10 is shown in Figure 10.

EMB10R Lines	Read by EMB10 Lines	EXPLANATION
100	4-5	Line 100 will always contain 14 items, regardless of problem complexity, as follows: NSS is the number of subspans into which the beam is naturally subdivided by the point loads and the discontinuities introduced by the distributed quantities. This is illustrated in Figure 7, where the finer, vertical lines denote the intervals. NOSUP is the number of support conditions to be examined. NMINT is the number of modes for which the eigenfrequency is desired.

		NMSHP($\leq$ NMINT) is the number of modes for which the shape and stress values are desired. NOINTP is a limiter for the number of interpolations to be performed in refining the eigenfrequency value (=10). G is the gravitational constant. BMLGTH is the total beam length, to be checked against SSUM, the sum of the NSS subspans. NPAR(I),I=1,7 are the numbers of the different values of the seven parameters PAR(I,J) described above. Thus there are NPAR(1)=4 different values of point weight, etc.
105-115	10-11	SS(I),I=1,NSS are the lengths of the subdivisions shown in Figure 7.
120	14-16	Line 120 will contain a minimum of 3 items up to an indeterminate maximum, depending on the program length limitations, as follows: NSUPP(I),I=1,NOSUP, are integer representations of the support conditions to be investigated (refer to Figure 8). MSHP(I),I=1,NMSHP are the modes for which the deflection and stress values are desired, in addition to eigenfrequency. (If no eigenmodes are desired, then MSHP(1=NMSHP)=0).
125-250	23-27	The loading, material and geometrical properties as shown in Figure 7 appear in single-column format together with the corresponding value of IPAR(I,J), which represents the number of the subdivision at which the property value occurs. Thus, the 3 lb. point load (=PAR(1,2)) appears at subdivision 7 (=IPAR(1,2)).

To verify that all input values have been read in from EMB10R properly, lines 6, 12, 17, 21 and 26 of EMB10 create the file MB10W1 as an echo check (Figure 12). EMB10 will be discussed with reference to the flow chart shown in Figure 11, which illustrates only the more involved steps in detail.

An error message (statement 153) will be printed out and operations will be terminated if SSUM, the total of SS(I),I=1,NSS is not equal to the value of BMLGTH as contained in EMB10R. This will afford an extra check on the values of SS(I) as stored in core during execution.

Statement 3 determines if a point load is present at the left-hand support, i.e. if IPAR(1,1)=1. If not, statements 3-6 add a zero mass at this point and shift the subscript J of PAR(I,J) and IPAR(I,J) up by unity. Lines 62-66 similarly detect a zero load at the right-hand support, necessitating PAR(1,NPAR1)=0.

The DO loops involving lines 67-85 distribute the continuous properties PAR(I,J),I=2,7 over the whole beam, giving NSS values of each property (refer to line 69). If, say, two or more different values of uniformly distributed load exist, the YES branch of line 70 is taken in order to assign the appropriate value of J such that PAR(2,J) is repositioned at the correct subspan. Thus, for the existing problem, PAR(2,4)(=3 lb/in) <u>as read in</u> must become PAR(2,10) and so forth. Subsequently these redistributed values must be spread out over the intermediate subspans, e.g. PAR(2,10) will be applied to subspans 11-14. This is accomplished by lines 77-85. Referring to file MB10W2 as shown in Figure 13, the property values as redistributed can be seen; also note that 17 (=NSS) values exist for each continuous property (I=2,7).

In order to establish an initial guess, to commence eigenfrequency computation in EMB15, an extreme range of ω_n is found in line 102. The lower bound, **Y**, is based on minimum stiffness and maximum distributed loading taken as continuous over the entire span and using a guided-free constraint. The upper bound, **X**, is based on maximum stiffness and minimum distributed loading, using

the clamped-clamped constraint. These values are entered in file MB10W3 as shown in Figure 14. Their natural logarithms are designated as FINCR, i.e. possible values for the initial increment. The employment of these values is completely discretionary.

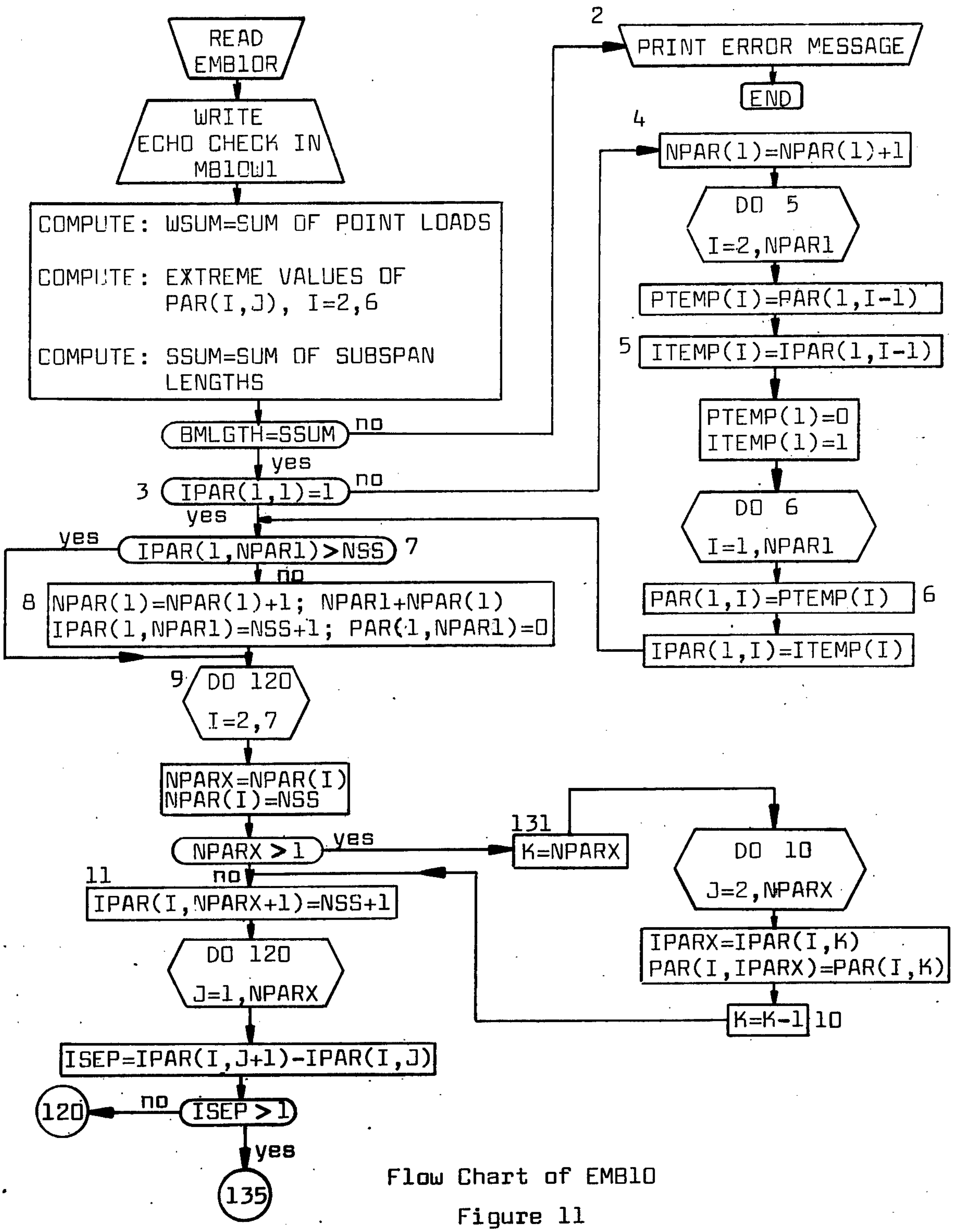
```
EMB10     13:53   FRI.  09/05/69.

$FILE EMB10R,MB10W1,MB10W2,MB10W3
DIMENSIØN NPAR(7),PAR(7,20),IPAR(7,20),PARMAX(6),PARMIN(6),MSHP(20),
+ ITEMP(20),PTEMP(20),SS(17),MINT(20),NSUPP(10)
READ (1,150) NSS,NØSUP,NMINT,NMSHP,NØINTP,G,BMLGTH,(NPAR(I),I=1,7)
150 FØRMAT(4X,5I5,2F9.3,7I3)
WRITE (2,160) NSS,NØSUP,NMINT,NMSHP,NØINTP,G,BMLGTH,(NPAR(I),I=1,7)
160 FØRMAT (29H      ECHØ CHECK ØF INPUT DATA/6X,4HNSS=,I3,3X,
+ 6HNØSUP=,I3,3X,6HNMINT=,I3,3X,6HNMSHP=,I3,3X,7HNØINTP=,I3/6X,
+ 2HG=,F8.3,3X,7HBMLGTH=,F8.3,13H  NPAR VALUES,7I3)
READ (1,151) (SS(I),I=1,NSS)
151 FØRMAT(4X,6F10.3)
WRITE (2,163) (SS(I),I=1,NSS)
163 FØRMAT(/6X,9HSS VALUES/(6X,6F10.3))
READ(1,152) (NSUPP(I),I=1,NØSUP),(MINT(I),I=1,NMINT),
+ (MSHP(I),I=1,NMSHP)
152 FØRMAT(4X,16I4)
WRITE (2,164) (NØSUP,(NSUPP(I),I=1,NØSUP)),
+ (NMINT,(MINT(I),I=1,NMINT)),(NMSHP,(MSHP(I),I=1,NMSHP))
164 FØRMAT (/6X,12HNSUPP VALUES/6X,*I4//6X,11HMINT VALUES
+ /6X,*I4//6X,11HMSHP VALUES/6X,*I4)
WRITE (2,168)
168 FØRMAT(/6X,19HPAR AND IPAR VALUES)
DØ 200 I=1,7
NPR=NPAR(I)
READ (1,154) (PAR(I,J),IPAR(I,J),J=1,NPR)
WRITE (2,165) (PAR(I,J),IPAR(I,J),J=1,NPR)
200 CØNTINUE
154 FØRMAT(4X,E11.4,I3)
165 FØRMAT(4(E11.4,I3))
ENDFILE 2
WSUM=0.
NPAR1=NPAR(1)
DØ 121 I=1,NPAR1
121 WSUM=WSUM+PAR(1,I)
DØ 122 I=2,6
NPARX=NPAR(I)
PARMAX(I)=PAR(I,1)
PARMIN(I)=PAR(I,1)
IF (IPAR(I,NPARX)-1) 12,122,12
12 DØ 122 J=2,NPARX
PARMIN(I)=AMIN1(PARMIN(I),PAR(I,J))
122 PARMAX(I)=AMAX1(PARMAX(I),PAR(I,J))
SSUM=0.
DØ 1 I=1,NSS
1 SSUM=SSUM+SS(I)
IF(ABS(BMLGTH-SSUM)-.0001)3,3,2
2 PRINT 153,SSUM
153 FØRMAT (54H INEQUALITY ØF BMLGTH AND CØMPUTED BEAMLENGTH,
+ LATTER=,F10.3)
GØTØ 99
3 IF(IPAR(1,1)-1)4,7,4
4 NPAR(1)=NPAR(1)+1
NPAR1=NPAR(1)
```

Program MEM810

Figure 10

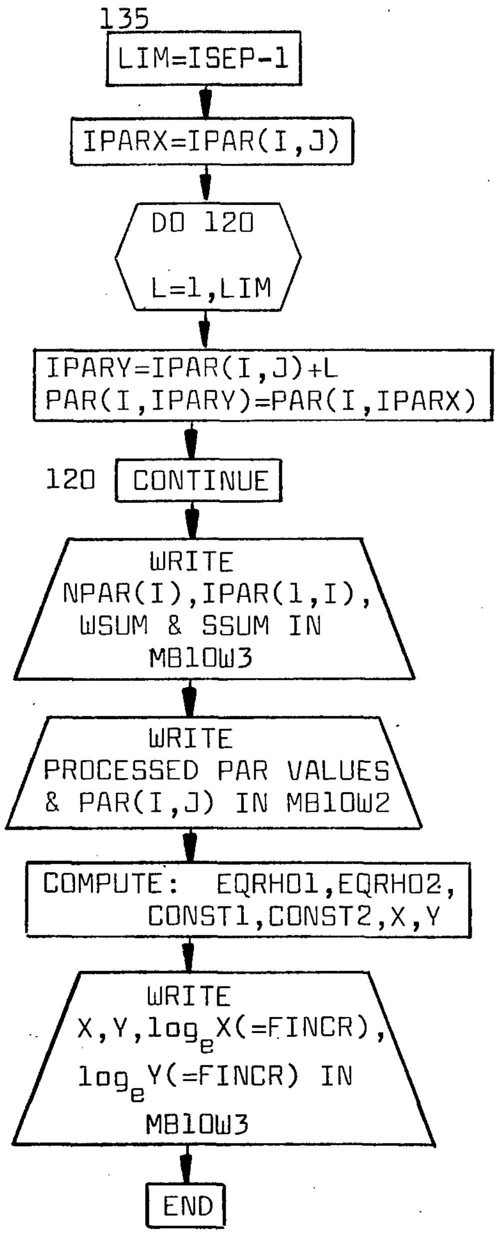
```
DØ 5 I=2,NPAR1
PTEMP(I)=PAR(1,I-1)
5 ITEMP(I)=IPAR(1,I-1)
PTEMP(1)=0.
ITEMP(1)=1
DØ 6 I=1,NPAR1
PAR(1,I)=PTEMP(I)
6 IPAR(1,I)=ITEMP(I)
7 IF(IPAR(1,NPAR1)-NSS)8,8,9
8 NPAR(1)=NPAR(1)+1
NPAR1=NPAR(1)
IPAR(1,NPAR1)=NSS+1
PAR(1,NPAR1)=0.
9 DØ 120 I=2,7
NPARX=NPAR(I)
NPAR(I)=NSS
IF (NPARX-1) 11,11,131
131 K=NPARX
DØ 10 J=2,NPARX
IPARX=IPAR(I,K)
PAR(I,IPARX)=PAR(I,K)
10 K=K-1
11 IPAR(I,NPARX+1)=NSS+1
DØ 120 J=1,NPARX
ISEP=IPAR(I,J+1)-IPAR(I,J)
IF(ISEP-1)120,120,135
135 LIM=ISEP-1
IPARX=IPAR(I,J)
DØ 120 L=1,LIM
IPARY=IPAR(I,J)+L
PAR(I,IPARY)=PAR(I,IPARX)
120 CØNTINUE
WRITE (4,161) (NPAR(I),I=1,7)
161 FØRMAT (6X,11HNPAR VALUES, 7I3)
WRITE (4,156) (NPAR1,(IPAR(1,I),I=1,NPAR1))
156 FØRMAT(/6X,11HIPAR VALUES/7X,*I4)
WRITE (4,157) WSUM,SSUM
157 FØRMAT (/6X,5HWSUM=,E15.8,7H  SSUM=,E15.8)
WRITE (3,169); 169 FØRMAT(5X,20HPRØCESSED PAR VALUES)
DØ 201 I=1,7
NPR=NPAR(I)
201 WRITE (3,155) (PAR(I,J),J=1,NPR)
155 FØRMAT(6X,4E15.8)
ENDFILE 3
EQRHØ1=(WSUM/SSUM+PARMIN(2)+PARMIN(3)*PARMIN(6))/G
EQRHØ2=(WSUM/SSUM+PARMAX(2)+PARMAX(3)*PARMAX(6))/G
CØNST1=(PARMAX(4)*PARMAX(5)/(EQRHØ1*SSUM**4))**.5
CØNST2=(PARMIN(4)*PARMIN(5)/(EQRHØ2*SSUM**4))**.5
WRITE(4,167) X=4.73^2*CØNST1,Y=1.571^2*CØNST2,LØG(X),LØG(Y)
167 FØRMAT(/54H       EXTREME RANGE ØF 1ST/2ND MØDE CIRCULAR
+ FREQUENCY/2E15.8/6HFINCR=,F7.4,11H  ØR FINCR=,F7.4)
ENDFILE 4
99 CØNTINUE
$ØPT SIZE
END
```

LENGTH
ABØUT 3300 CHARS.



Flow Chart of EMB10

Figure 11


135
LIM=ISEP-1
IPARX=IPAR(I,J)
DO 120
L=1,LIM
IPARY=IPAR(I,J)+L
PAR(I,IPARY)=PAR(I,IPARX)
120
CONTINUE
WRITE
NPAR(I),IPAR(1,I),
WSUM & SSUM IN
MB10W3
WRITE
PROCESSED PAR VALUES
& PAR(I,J) IN MB10W2
COMPUTE: EQRHO1,EQRHO2,
CONST1,CONST2,X,Y
WRITE
X,Y,$\log_e$X(=FINCR),
$\log_e$Y(=FINCR) IN
MB10W3
END

```
MB10W1      17:42    WED.  09/03/69.

     ECHØ CHECK ØF INPUT DATA
      NSS= 17    NØSUP=  3    NMINT=  5    NMSHP=  5    NØINTP= 10
      G= 386.000   BMLGTH=  36.000  NPAR VALUES  4  5  4  4  3  3  3

      SS VALUES
           3.400       1.700       2.100       2.400       1.400        .900
           3.000       3.400        .800       2.700       2.100       2.100
           1.100       3.100       1.900       1.700       2.200

      NSUPP VALUES
         2   7  10

      MINT VALUES
         1   2   3   4   5

      MSHP VALUES
         1   2   3   4   5

      PAR AND IPAR VALUES
  .7000E+01  3  .3000E+01  7  .4000E+01 11  .6000E+01 16
  .0000E+01  1  .2000E+01  2  .0000E+01  5  .3000E+01 10
  .0000E+01 15
  .1000E+00  1  .3000E+00  4  .2000E+00  9  .5000E-01 17
  .3000E+08  1  .3500E+08  6  .1000E+08 12  .3300E+08 14
  .4500E-03  1  .5210E-02  8  .0125E-03 13
  .6000E-01  1  .2500E+00  8  .1500E-01 13
  .1500E+00  1  .2500E+00  8  .5000E-01 13
```

Permanent File MB10W1

Figure 12

MB10W2 13:39 THUR. 08/14/69.

PROCESSED PAR VALUES

PAR(1,J)	.00000000E+01	.70000000E+01	.30000000E+01	.40000000E+01
	.60000000E+01	.00000000E+01		
PAR(2,J)	.00000000E+01	.20000000E+01	.20000000E+01	.20000000E+01
	.00000000E+01	.00000000E+01	.00000000E+01	.00000000E+01
	.00000000E+01	.30000000E+01	.30000000E+01	.30000000E+01
	.30000000E+01	.30000000E+01	.00000000E+01	.00000000E+01
	.00000000E+01			
PAR(3,J)	.10000000E+00	.10000000E+00	.10000000E+00	.30000000E+00
	.30000000E+00	.30000000E+00	.30000000E+00	.30000000E+00
	.20000000E+00	.20000000E+00	.20000000E+00	.20000000E+00
	.20000000E+00	.20000000E+00	.20000000E+00	.20000000E+00
	.50000000E-01			
PAR(4,J)	.30000000E+08	.30000000E+08	.30000000E+08	.30000000E+08
	.30000000E+08	.35000000E+08	.35000000E+08	.35000000E+08
	.35000000E+08	.35000000E+08	.35000000E+08	.10000000E+08
	.10000000E+08	.33000000E+08	.33000000E+08	.33000000E+08
	.33000000E+08			
PAR(5,J)	.45000000E-03	.45000000E-03	.45000000E-03	.45000000E-03
	.45000000E-03	.45000000E-03	.45000000E-03	.52100000E-02
	.52100000E-02	.52100000E-02	.52100000E-02	.52100000E-02
	.01250000E-03	.01250000E-03	.01250000E-03	.01250000E-03
	.01250000E-03			
PAR(6,J)	.60000000E-01	.60000000E-01	.60000000E-01	.60000000E-01
	.60000000E-01	.60000000E-01	.60000000E-01	.25000000E+00
	.25000000E+00	.25000000E+00	.25000000E+00	.25000000E+00
	.15000000E-01	.15000000E-01	.15000000E-01	.15000000E-01
	.15000000E-01			
PAR(7,J)	.15000000E+00	.15000000E+00	.15000000E+00	.15000000E+00
	.15000000E+00	.15000000E+00	.15000000E+00	.25000000E+00
	.25000000E+00	.25000000E+00	.25000000E+00	.25000000E+00
	.50000000E-01	.50000000E-01	.50000000E-01	.50000000E-01
	.50000000E-01			

Permanent File MB10W2

Figure 13

MB10W3 17:44 WED. 09/03/69.

NPAR VALUES 6 17 17 17 17 17 17

IPAR VALUES
1 3 7 11 16 18

WSUM= .20000000E+02 SSUM= .36000000E+02

EXTREME RANGE ØF 1ST/2ND MØDE CIRCULAR FREQUENCY
.19418117E+03 .21953789E+00
FINCR= 5.2688 ØR FINCR=-1.5162

Permanent File MB10W3

Figure 14

(This page left intentionally blank)

Computation of Eigenfrequencies and State Vectors

The saved items EMB15/20/21 accept the data conditioned by EMB10 and compute the eigenfrequencies and state vectors for each MINT(I) and NSUPP(II) without interruption until the problem is completed. If eigenmodes or further refinement of particular eigenfrequencies are desired, however, execution may be terminated. For efficiency, solutions may begin at any value of guessed frequency and any mode at the discretion of the user. Array storage requirements have been minimized by this restart feature; thus when 30 values of frequency FREQ(I) have been computed and used to determine the corresponding frequency function DIF(I) and more are desired, execution is recommenced by the user with the index I reset to 1 and employing the previous frequency as FREQ(1). If the current MINT=MSHP(IN), i.e. the eigenmode is desired, the required data corresponding to the eigenfrequency as computed after ten interpolations are entered into file MB15W1 and linked file MB15W2/MB15W3 by EMB20 and the message "RUN EMB25 FOR MODAL VALUES" is printed.

The flow chart of EMB15 is shown in Figure 16 and the program itself appears as Figure 15. The input is largely from permanent files EMB10R, MB10W2 and MB10W3. Terminal-supplied input (lines 48-50) is confined to values which implement the restart feature.

Files	Read by EMB15 Lines	EXPLANATION
EMB10R	10-22	Refer to "Conditioning of Input Data".
MB10W2	30-38	Refer to Figure 13.
MB10W3	24-26	This input furnishes the number of point loads to be considered, including virtual loads, if any. (NPAR(I),I=2,7 are all =NSS)

MB10W3	40-42	The subspans corresponding to each point load, IPAR(1,J) are needed for the computation of point transfer matrices.

Initially and when reentry into the program is attempted a message is printed requesting seven parameters from the terminal:
1) II is the index of the support condition NSUPP and can be changed at will to allow, for example, an investigation for eigenvalues within a given frequency range for various end constraints.
2) IM & IN are the subscripts of MINT and MSHP respectively and can be set independently to allow the determination of an eigenfrequency of interest for other than MINT(1).
3) NMODE denotes the mode currently being investigated.
4) FREQ(1) is the initial frequency to be used as a starting point in the spectrum under examination.
5) FINCR is the natural logarithmic increment of frequency to be added to FREQ(1) and may be either arbitrary or drawn from the extreme range of values as computed by EMB10. Negative values must be avoided.
6) IND(I) indicates the polarity of the frequency function DIF(I) (=R(ω)); i.e. IND(I)=1 if DIF(I) > 0, otherwise IND(I)=2. IND(I) is needed to initialize the procedure in EMB20 for detecting polarity reversal, i.e. if (IND(I)-IND(I-1)). The user will initially not know the sign of DIF(1) and thus a restart may be indicated if the wrong choice is made.

In line 42 an arithmetic statement function is defined which represents the procedure for interpolating between the previous frequency, FREQ(I-1) and FREQ(ICROSS), the last frequency corresponding to a DIF value of different sign.

In lines 52-54 the frequency-independent B(J),J=1,NSS are computed for use later in establishing field transfer matrices. Lines 58-68 set FRQN(1)=0 for the first four support conditions and increment IM and IN to 2 (if MINT(1)=1 through oversight). The SS(J) values have not been included in B(J) as an expedient for

saved item EMB25, which reads the COEFM(J) and further subdivides the subspans; thus it is desirable that T (=COEFM(J)=B(J)*FREQ(I)**½) does not involve SS(J) at this stage. If, at line 56, NMODE $\geq$ 2, the immediately preceding eigenfrequency will be supplied at the terminal. This is utilized by EMB20 to compute a FINCR for the ensuing mode, i.e. for NMODE+1.

In line 72, FREQ(2) is determined exponentially; arithmetic incrementing would be less efficient in the preliminary detection of eigenvalues. Care should be taken not to supply FRQN(1)=0 to avoid an execution error in connection with the intrinsic LOG function.

Statement 16 is a computed GO TO which initializes the first state column vector STAMAT and correction column vector CORMAT according to the support condition. Thus for the guided-free constraint (NSUPP(1)=2) the initial vectors are:

$$\begin{aligned} \text{STAMAT}_1 &= \{1\ 1\ 0\ 0\} \\ \text{CORMAT}_1 &= \{0\ 1\ 0\ 0\} \end{aligned} \tag{16}$$

Here the free-end deflection is normalized to one inch and the slope chosen as unity. Lines 84-128 establish the transfer matrices, indices K and J being incremented at each point load and elastic field respectively. Figure 24 illustrates how the K and J values are assigned for this beam. Whenever J=IPAR(1,K) a point transfer matrix is computed in lines 88-90. If, subsequently, NSS < IPAR(1,K), (line 92) then matrix computation is completed for FREQ(I) and control passes to EMB21 to obtain the state vectors and correction factor by chain multiplication; otherwise computation of the Jth field matrix is indicated—according to Figure 3 if the field is effectively massless (i.e. if PAR(3,J) $\leq$ .0001) or Figure 2 otherwise. The argument of the natural trigonometric and hyperbolic functions assumes its largest value at SS(13), i.e. $ARG_{max} = .097724\,\omega^{1/2}$ and the largest permissible argument = |176|, which corresponds to ω_{max} = 517 kHz approximately. Argument magnitude difficulties may be present with beams having longer and more compliant elastic fields. In such cases a virtual load could be inserted at midspan, thus halving the argument.

For example, the natural frequencies in pure bending of a 1" square uniform aluminum beam 25" long and clamped at each end, as reported in Reference 2, were corroborated to nearly 27 000 rad/sec (until the argument exceeded |176|).

EMB21 appears as Figure 17 and is charted in Figure 18. Its function, as an item which is compiled with EMB20 and EMB25, is to obtain the intermediate and final state vectors (STAMAT(J,K,1), J=2,NSS+NPAR(1)+1),K=1,4 and the elements of the correction vector CORMAT(J,K,1). At each FREQ(I) as mentioned previously two iterations are needed to obtain DIF(I); ITER=0 causes both vectors to be computed in lines 154-160. If ITER=1 only STAMAT is recomputed. By the computed GO TO in line 162 the components COR1 and COR2 of the correction term COR are obtained in accordance with the support condition. Thus for this beam slope and shear force at the right-hand support are ideally =0, causing M=2 and N=4. COR1 and COR2 are obtained from the relationship

$$\text{STAMAT(LIM+1,2,1)+COR1*CORMAT(LIM+1,2,1)=0} = \Psi_{m+n+1} \qquad (17)$$

$$\text{STAMAT(LIM+1,4,1)+COR2*CORMAT(LIM+1,4,1)=0} = V_{m+n+1} \qquad (18)$$

The correction term COR is taken as the arithmetic mean of COR1 and COR2 and is applied in statement 105 to the left-hand slope, the other elements of Z_1 remaining the same. ITER is now made =1 and statement 300 is reentered. After the recomputation of STAMAT, exit is made to statement 119 in either EMB20 or EMB25 as appropriate. In EMB20 (refer to Figures 19 and 20) statement 119 computes the frequency function as the difference between the components of Z_{m+n+1} (which should be zero at the eigenfrequency). According to {i} this difference is to be taken as COR1-COR2, but this involves magnitudes that become progressively smaller as NMODE increases. This results from the growth of the CORMAT elements with NMODE, causing the graph of DIF vs. FREQ to flatten out. In some cases this may hinder eigenfrequency determination if sufficiently severe.

Statement 80 causes control to branch to 84 if interpolation is not being performed, i.e. if the frequency axis has not yet been crossed for the current NMODE=MINT(IM). The test for polarity reversal of DIF(I) is statement 84—the YES branch being taken to obtain additional DIF values until a reversal does occur, whereupon the current I is retained in MAX to preserve IND(I) and also allow frequency incrementing to commence with FREQ(MAX) after all interpolations are completed. The first approximation to FRQN(NMODE) is a linear interpolation between FREQ(I) and FREQ(I-1) and is printed by lines 21 and 23. After this first axis crossing, if NMODE=MINT(IM) the YES branch is taken to 97 and ten (=NOINTP) further interpolations are performed to refine FRQN. I-1 is retained in ICROSS to preserve the FREQ value previous to the axis crossing. The first approximation to FRQN(NMODE) is put into FREQ(I+1) and control is returned to statement 16 in EMB15 to obtain the corresponding DIF. Interpolation proceeds until INTERP≥NOINTP, at which point the final value of FRQN is equated to the current FREQ. If a sign change in DIF occurs during interpolation and INTERP < NOINTP, then lines 37 and 39 assign the current I-1 to ICROSS and control is returned to EMB15.

If the current eigenfrequency was the last to be determined, i.e. NMODE=MINT(NMINT), then statement 133 determines if the eigenmode is also required, i.e. if NMODE=MSHP(NMSHP). The NO branch from 133 leads to the question of whether or not all support conditions have been investigated, i.e. if NOSUP > II. Here the NO branch indicates problem completion, a notice to this effect being printed by statement 134; otherwise 139 causes a request to be printed for the new NSUPP, and control is returned to input statement 12 of EMB15. The YES branch from 133 causes all data needed by EMB25 to be entered into permanent files MB15W1 and MB15W2/MB15W3 by lines 79-99. This involves the state vectors STAMAT and all transfer matrices BMAT; format statements 75, 76, 78 and 79 arrange the elements to facilitate visualization of the matrix multiplication (refer to Figure 21 which shows the first five state vectors for the fourth eigenfrequency of 156.7256 rad/sec (=24.944 Hz)). MB15W1 is shown

in Figure 22. FRQN(NMODE) is added for completeness and FINCR will be required when EMB15 is rerun.

Lines 59-77 establish the new increment for the succeeding NMODE on the basis of support condition and the value of NMODE. Thus for NMODE=1 or 2 and NATSP=1,2,3 or 4 there exists no FRQN(NMODE-1) as required by statement 72, causing FINCR=.1. Otherwise statement 72 assigns a value to FINCR based on the log ratio of the current to the previous FRQN if NMODE=2; if NMODE=1 for NATSP=5-10 then FINCR=.1. After FINCR has been determined by statement 70 or 72 it is recorded in MB15W1 if the eigenmode is desired, otherwise line 73 increments I, IM and NMODE. The IND(I-1) and FREQ(I) values are based on IND(MAX) and FREQ(MAX).

Figure 23 shows the results for NMODE=NATSP=2. The sensitivity of DIF to FREQ is quite apparent at I=19 and 20, where 9th and 10th place differences in FREQ cause an 80.2% decrease in DIF.

(This page left intentionally blank)

```
EMB15      13:24    WED.   09/03/69.

$FILE EMB10R,MB10W2,MB10W3,MB15W1,MB15W2/MB15W3
DIMENSIØN BMAT(23,4,4),CØEFM(17),FREQ(30),DIF(30),IND(30),
+ FRQN(30),MINT(30),MSHP(30),NSUPP(10),NPAR(7),SS(17),
+ PAR(7,20),IPAR(1,20),B(17),STAMAT(24,4,1),CØRMAT(24,4,1)
READ (1,290) NSS,NØSUP,NMINT,NMSHP,NØINTP,G
290 FØRMAT (4X,5I5,F9.3)
READ (1,292) (SS(I),I=1,NSS)
292 FØRMAT (4X,6F10.3)
READ (1,291) (NSUPP(I),I=1,NØSUP),(MINT(I),I=1,NMINT),
+ (MSHP(I),I=1,NMSHP)
291 FØRMAT (4X,16I4)
READ(3,293) (NPAR(I),I=1,7)
293 FØRMAT (17X,7I3)
READ(2)
DØ 294 I=1,7
NPR=NPAR(I)
NPAR1=NPAR(1)
294 READ (2,295) (PAR(I,J),J=1,NPR)
295 FØRMAT (6X,4E15.8)
READ(3,297) (NPAR1,(IPAR(1,I),I=1,NPAR1))
297 FØRMAT(//7X,*I4); R(I)=-ABS(DIF(I-1))/(ABS(DIF(I-1))
+ +ABS(DIF(ICRØSS)))*(FREQ(I-1)-FREQ(ICRØSS))+FREQ(I-1)
LIM=NSS+NPAR(1); 12 PRINT,"INPUT VALUES ØF II,IM,IN,NMØDE,FREQ(1),
+FINCR AND IND(1)"; INPUT,II,IM,IN,NMØDE,FREQ(1),FINCR,IND(1)
NATSP=NSUPP(II); ITER=INTERP=0; I=1
DØ 11 J=1,NSS; 11 B(J)=((PAR(2,J)+PAR(3,J)*PAR(6,J))/
+ (G*PAR(4,J)*PAR(5,J)))**.25
IF(NMØDE-1) 201,201,202; 202 PRINT 203,"FRQN(",NMØDE-1,")="
203 FØRMAT(I2); INPUT, FRQN(NMØDE-1); GØ TØ 15
201 GØ TØ(13,13,13,13,15,15,15,15,15,15), NATSP
13 FRQN(1)=0.
NMØDE=2
IF(MINT(1)-1) 15,200,15
200 IM=IN=2
15 I=I+1
FREQ(I)=EXP(LØG(FREQ(I-1))+FINCR)
16 GØ TØ (25,25,25,26,26,28,29,28,28,28), NATSP
25 STAMAT(1,1,1)=STAMAT(1,2,1)=1.; GØ TØ 27
26 STAMAT(1,1,1)=STAMAT(1,3,1)=1.; GØ TØ 32
28 STAMAT(1,3,1)=STAMAT(1,4,1)=1.; GØ TØ 32
29 STAMAT(1,2,1)=STAMAT(1,4,1)=1.; 27 CØRMAT(1,2,1)=1.; GØ TØ 39
32 CØRMAT(1,3,1)=1.; 39 K=1; J=0; 17 J=J+1
IF(J-IPAR(1,K)) 23,20,23
20 DØ 22 L=1,4; 22 BMAT(J+K-1,L,L)=1.; BMAT(J+K-1,4,1)=PAR(1,K)
+ *FREQ(I)**2/G
IF(NSS-IPAR(1,K)) 24,19,19; 19 K=K+1
23 N=J+K-1; EI=PAR(4,J)*PAR(5,J); IF(PAR(3,J)-.0001) 5,5,6
5 CØEFM(J)=0.; DØ 2 L=1,4; DØ 1 M=1,4; 1 BMAT(N,L,M)=0.
2 BMAT(N,L,L)=1.; BMAT(N,1,2)=BMAT(N,3,4)=SS(J)
BMAT(N,1,3)=BMAT(N,2,4)=SS(J)**2/(2.*EI); BMAT(N,2,3)=SS(J)/EI
BMAT(N,1,4)=SS(J)**3/(6.*EI); GØ TØ 17
```

Saved Item EMB15

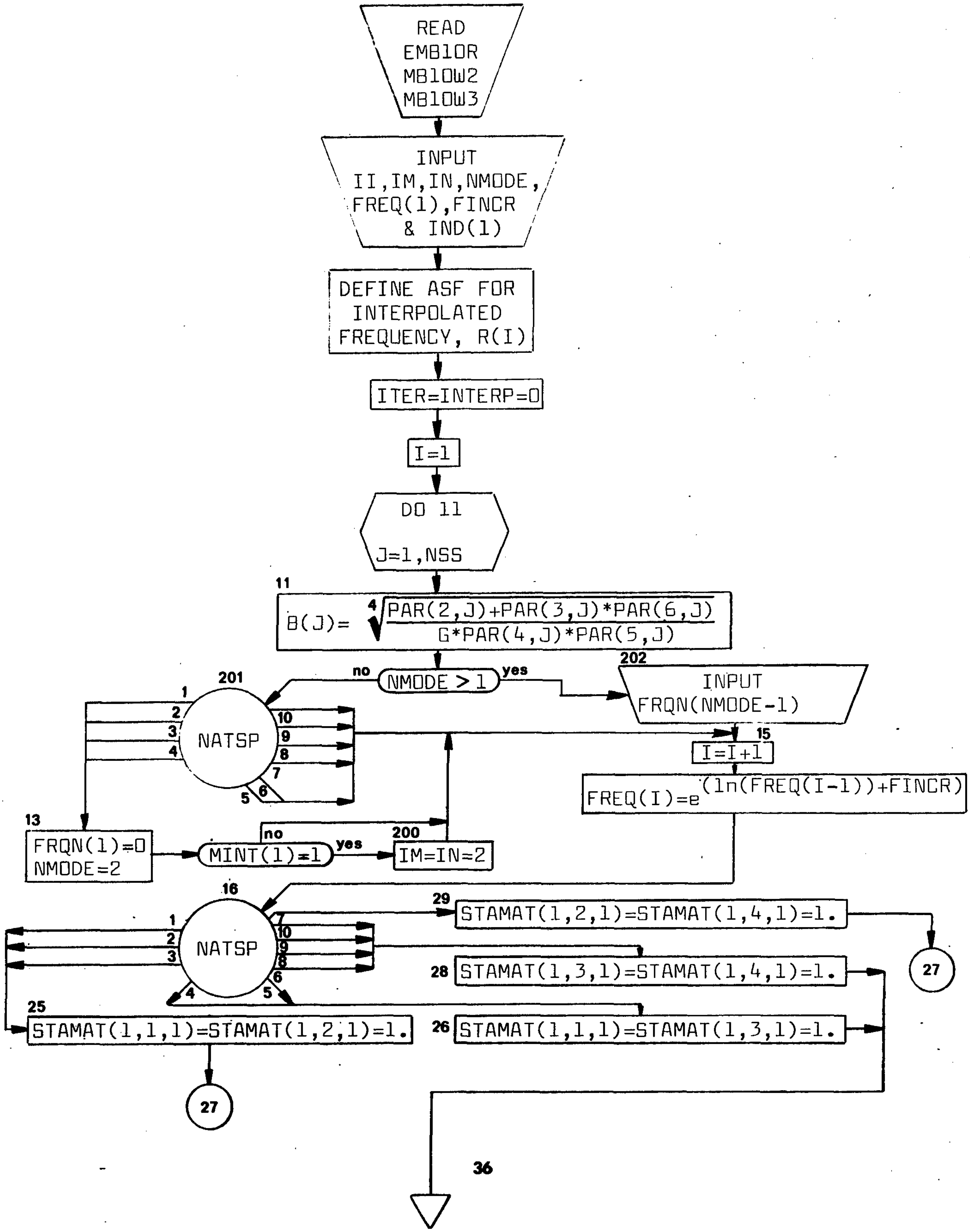
Figure 15

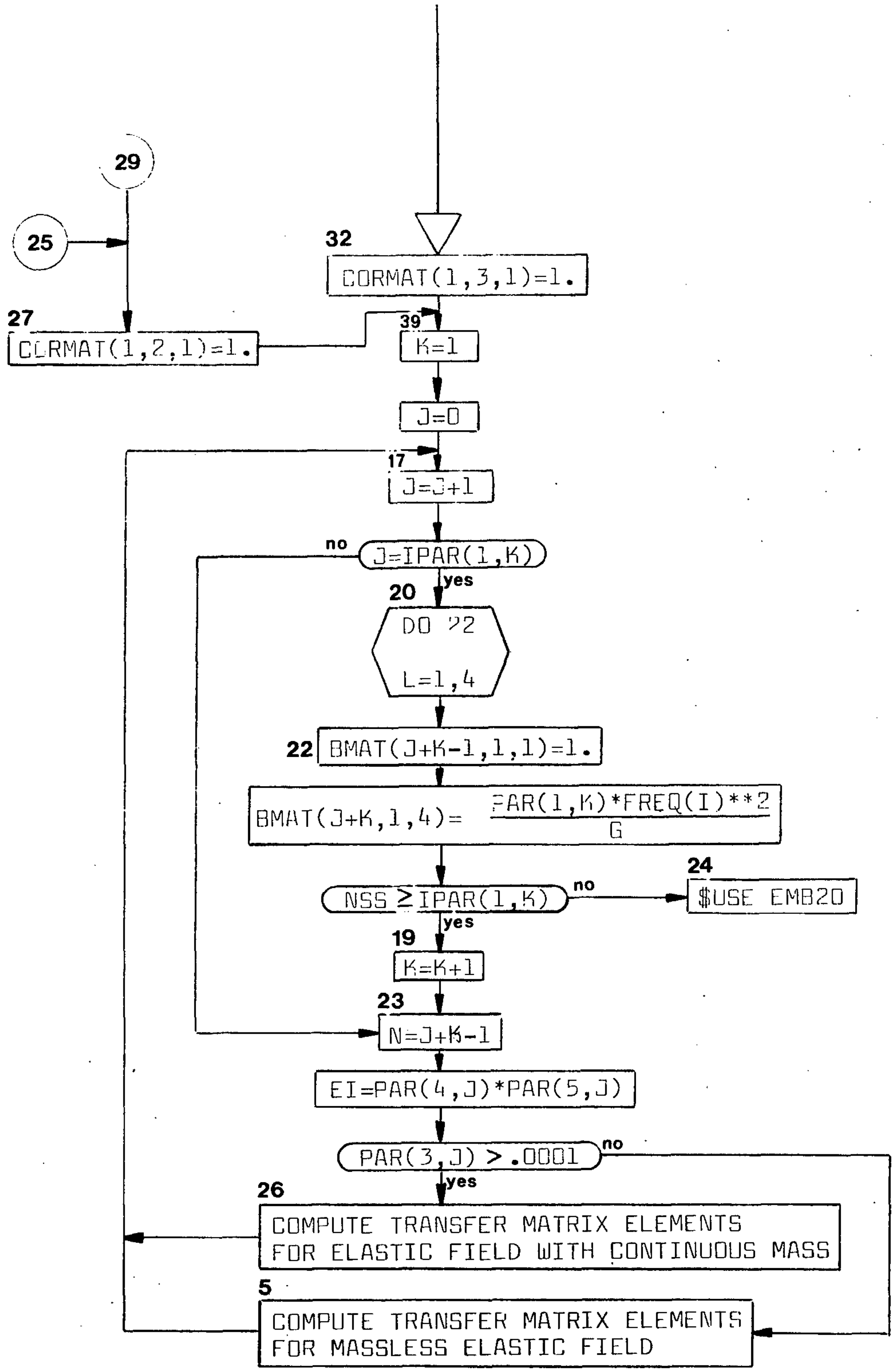
```
6 T=CØEFM(J)=B(J)*FREQ(I)**.5; ARG=T*SS(J)
U=(EXP(ARG)+EXP(-ARG))/2.; V=CØS(ARG); W=U-V
X=(EXP(ARG)-EXP(-ARG))/2.; Y=SIN(ARG); Z=X-Y
BMAT(N,1,1)=BMAT(N,2,2)=BMAT(N,3,3)=BMAT(N,4,4)=(U+V)/2.
BMAT(N,1,2)=BMAT(N,3,4)=(X+Y)/(2.*T)
BMAT(N,1,3)=BMAT(N,2,4)=W/(2.*EI*T**2)
BMAT(N,1,4)=Z/(2.*EI*T**3)
BMAT(N,2,1)=BMAT(N,4,3)=T*Z/2.
BMAT(N,2,3)=(X+Y)/(2.*EI*T)
BMAT(N,3,1)=BMAT(N,4,2)=EI*T**2*Z/2.
BMAT(N,3,2)=EI*T*Z/2.
BMAT(N,4,1)=EI*T**3*(X+Y)/2.
GØ TØ 17
24 $USE EMB20
```

LENGH
ABØUT 3000 CHARS.

Flow Chart of EMB15

Figure 16




29
25
32
CORMAT(1,3,1)=1.
27
CORMAT(1,2,1)=1.
39
K=1
J=0
17
J=J+1
no
J=IPAR(1,K)
yes
20
DO 22
L=1,4
22
BMAT(J+K-1,1,1)=1.
BMAT(J+K,1,4)= PAR(1,K)*FREQ(I)**2 / G
24
NSS ≥ IPAR(1,K)
no
$USE EMB20
yes
19
K=K+1
23
N=J+K-1
EI=PAR(4,J)*PAR(5,J)
PAR(3,J) > .0001
no
yes
26
COMPUTE TRANSFER MATRIX ELEMENTS
FOR ELASTIC FIELD WITH CONTINUOUS MASS
5
COMPUTE TRANSFER MATRIX ELEMENTS
FOR MASSLESS ELASTIC FIELD

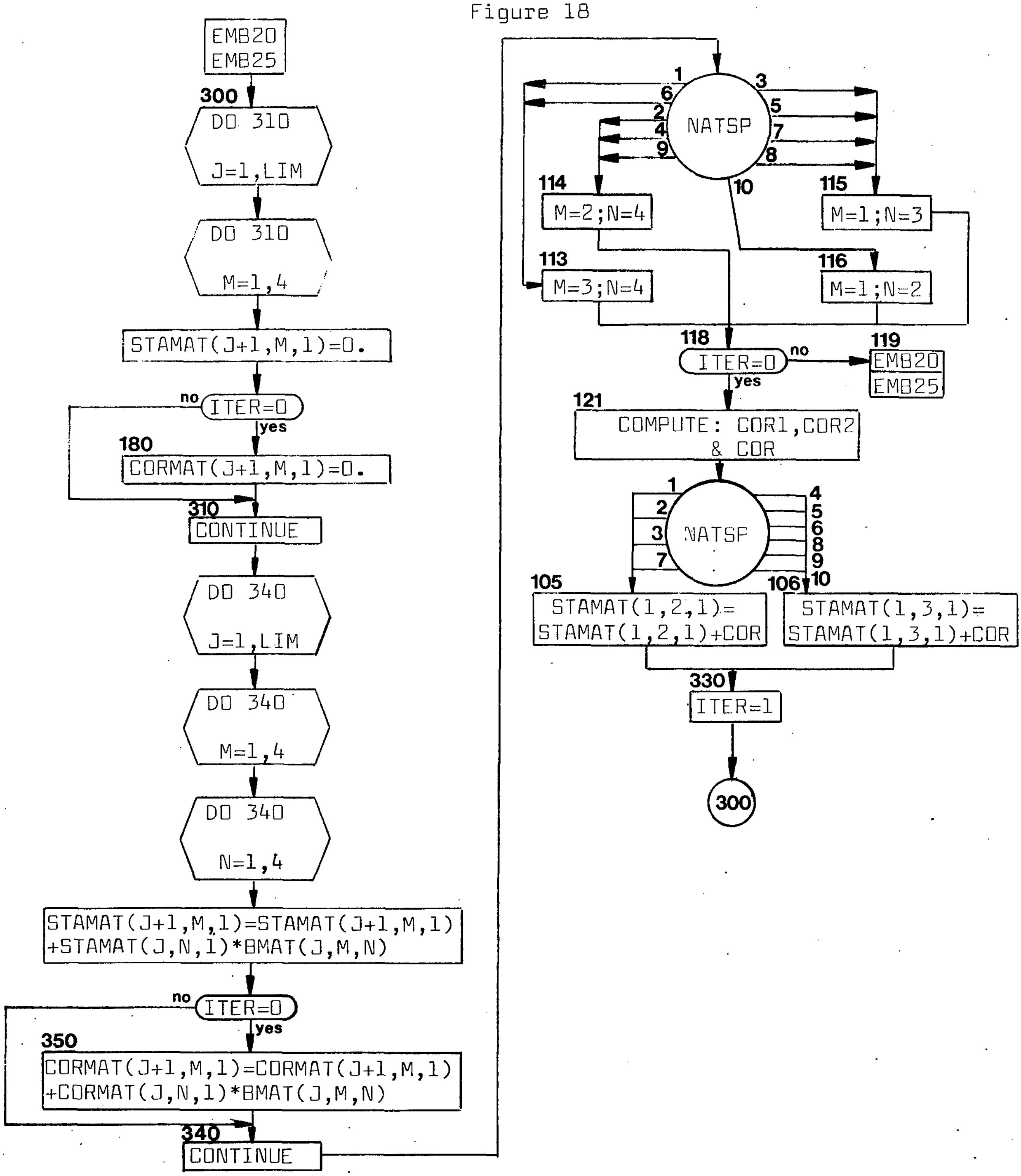
```
EMB21       4:38    SAT.  09/06/69.

300 DØ 310 J=1,LIM; DØ 310 M=1,4; STAMAT(J+1,M,1)=0.
IF(ITER) 310,180,310; 180 CØRMAT(J+1,M,1)=0.; 310 CØNTINUE
DØ 340 J=1,LIM; DØ 340 M=1,4; DØ 340 N=1,4
STAMAT(J+1,M,1)=STAMAT(J+1,M,1)+STAMAT(J,N,1)*BMAT(J,M,N)
IF(ITER) 340,350,340; 350 CØRMAT(J+1,M,1)=CØRMAT(J+1,M,1)+
+ CØRMAT(J,N,1)*BMAT(J,M,N); 340 CØNTINUE
GØ TØ (113,114,115,114,115,113,115,115,114,116), NATSP
113 M=3; N=4; GØ TØ 118
114 M=2; N=4; GØ TØ 118
115 M=1; N=3; GØ TØ 118
116 M=1; N=2; 118 IF(ITER) 119,121,119; 121 CØR1=
+ -STAMAT(LIM+1,M,1)/CØRMAT(LIM+1,M,1)
CØR2=-STAMAT(LIM+1,N,1)/CØRMAT(LIM+1,N,1)
CØR=(CØR1+CØR2)/2.
GØ TØ (105,105,105,106,106,106,105,106,106,106),NATSP
105 STAMAT(1,2,1)=STAMAT(1,2,1)+CØR; GØ TØ 330
106 STAMAT(1,3,1)=STAMAT(1,3,1)+CØR; 330 ITER=1; GØ TØ 300
```

Saved Item EMB21

Figure 17

Flow Chart of EMB21

Figure 18



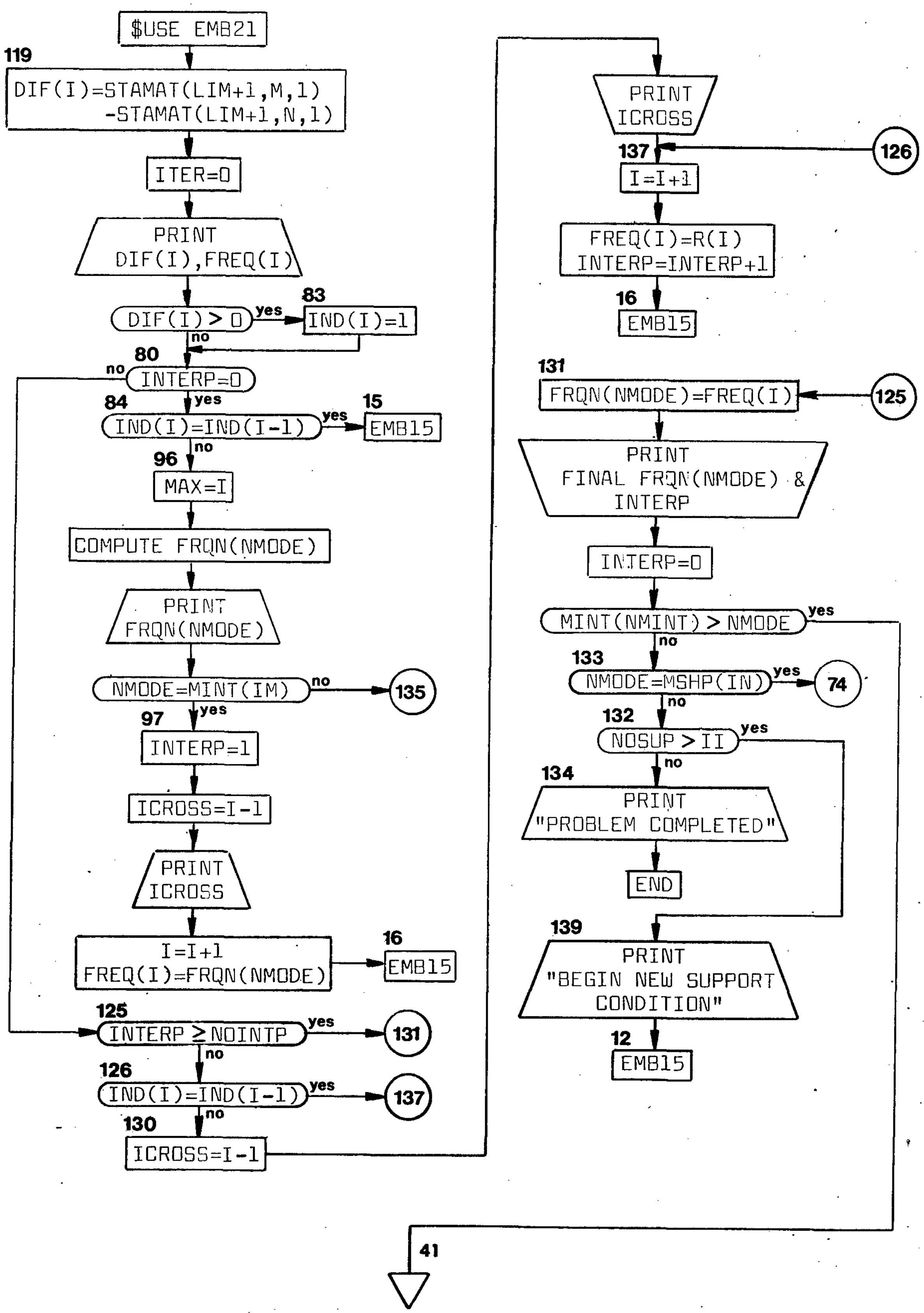
```
EMB20     18:02   MØN.  08/18/69.

$USE EMB21
119 DIF(I)=STAMAT(LIM+1,M,1)-STAMAT(LIM+1,N,1); ITER=0
PRINT 401, "DIFFERENCE VALUE(",I,")=",DIF(I)," FREQ=",FREQ(I)
401 FØRMAT(I2,2E15.8)
IF(DIF(I)) 81,81,83
81 IND(I)=2; GØ TØ 80; 83 IND(I)=1
80 IF(INTERP) 125,84,125
84 IF(IND(I-1)-IND(I)) 96,15,96
96 MAX=I; FRQN(NMØDE)=ABS(DIF(I-1))/(ABS(DIF(I))+ABS(DIF(I-1)))
+ *(FREQ(I)-FREQ(I-1))+FREQ(I-1)
PRINT 402,^"RØUGH FRQN(",NMØDE,")=",FRQN(NMØDE),"= ",
+ FRQN(NMØDE)/6.2832,"HZ",^^; 402 FØRMAT(I2,2E15.8)
IF(NMØDE-MINT(IM)) 135,97,135
97 INTERP=1; ICRØSS=I-1; PRINT 403, ICRØSS; I=I+1
FREQ(I)=FRQN(NMØDE); GØ TØ 16
403 FØRMAT(7HICRØSS=,I4/)
125 IF(NØINTP-INTERP) 131,131,126
126 IF(IND(I)-IND(I-1)) 130,137,130
130 ICRØSS=I-1; PRINT 403,ICRØSS; 137 I=I+1; FREQ(I)=R(I)
INTERP=INTERP+1; GØ TØ 16
131 FRQN(NMØDE)=FREQ(I)
PRINT 405,^"FINAL FRQN(",NMØDE,")=",FRQN(NMØDE),"= ",FRQN(NMØDE)
+ /6.2832,"HZ"," INTERP=",INTERP; 405 FØRMAT(I2,2E15.8,I3//)
INTERP=0
IF(MINT(NMINT)-NMØDE) 133,133,135; 133 IF(NMØDE-MSHP(IN))132,74,132
132 IF(NØSUP-II) 134,134,139; 139 PRINT,"BEGIN NEW SUPPØRT
+ CØNDITIØN"; GØ TØ 12
134 PRINT,"PRØBLEM CØMPLETED"; GØ TØ 304
135 IF(NMØDE-2) 136,136,72
136 GØ TØ (70,70,70,70,71,71,71,71,71,71), NATSP
70 FINCR=.1
IF(NMØDE-MSHP(IN)) 73,74,73
71 GØ TØ (70,72),NMØDE
72 FINCR=.1*LØG(FRQN(NMØDE)/FRQN(NMØDE-1))
IF(NMØDE-MSHP(IN)) 73,74,73
73 I=I+1; IM=IM+1; IND(I-1)=IND(MAX); NMØDE=NMØDE+1
FREQ(I)=EXP(LØG(FREQ(MAX))+FINCR)
GØ TØ 16
74 WRITE (5,75); 75 FØRMAT(59X,9HSTAMAT(1))
WRITE (5,76) (STAMAT(1,M,1),M=1,4); 76 FØRMAT (57X,E14.8)
DØ 77 J=1,LIM; WRITE (5,78) J,J+1
78 FØRMAT(/5HBMAT(,I2,1H),51X,7HSTAMAT(,I2,1H))
DØ 77 K=1,4; WRITE (5,79) (BMAT(J,K,L),L=1,4),STAMAT(J+1,K,1)
79 FØRMAT(4E14.8,X,E14.8); 77 CØNTINUE; ENDFILE 6
WRITE(4,69) NMØDE,MINT(NMINT),II,NATSP,FRQN(NMØDE),
+ (CØRMAT(1,M,1),M=1,4),(J,CØEFM(J),J=1,NSS)
69 FØRMAT(6HNMØDE=,I2,14H  MINT(NMINT)=,I2,5H  II=,I2,
+ 8H  NATSP=,I2,7H  FRQN=,E15.8/9HCØRMAT(1),4F4.1
+ /(6HCØEFM(,I2,3H)= ,E15.8)); ENDFILE 4
PRINT,"RUN EMB25 FØR MØDAL VALUES"
304 CØNTINUE
$ØPT SIZE
END
```

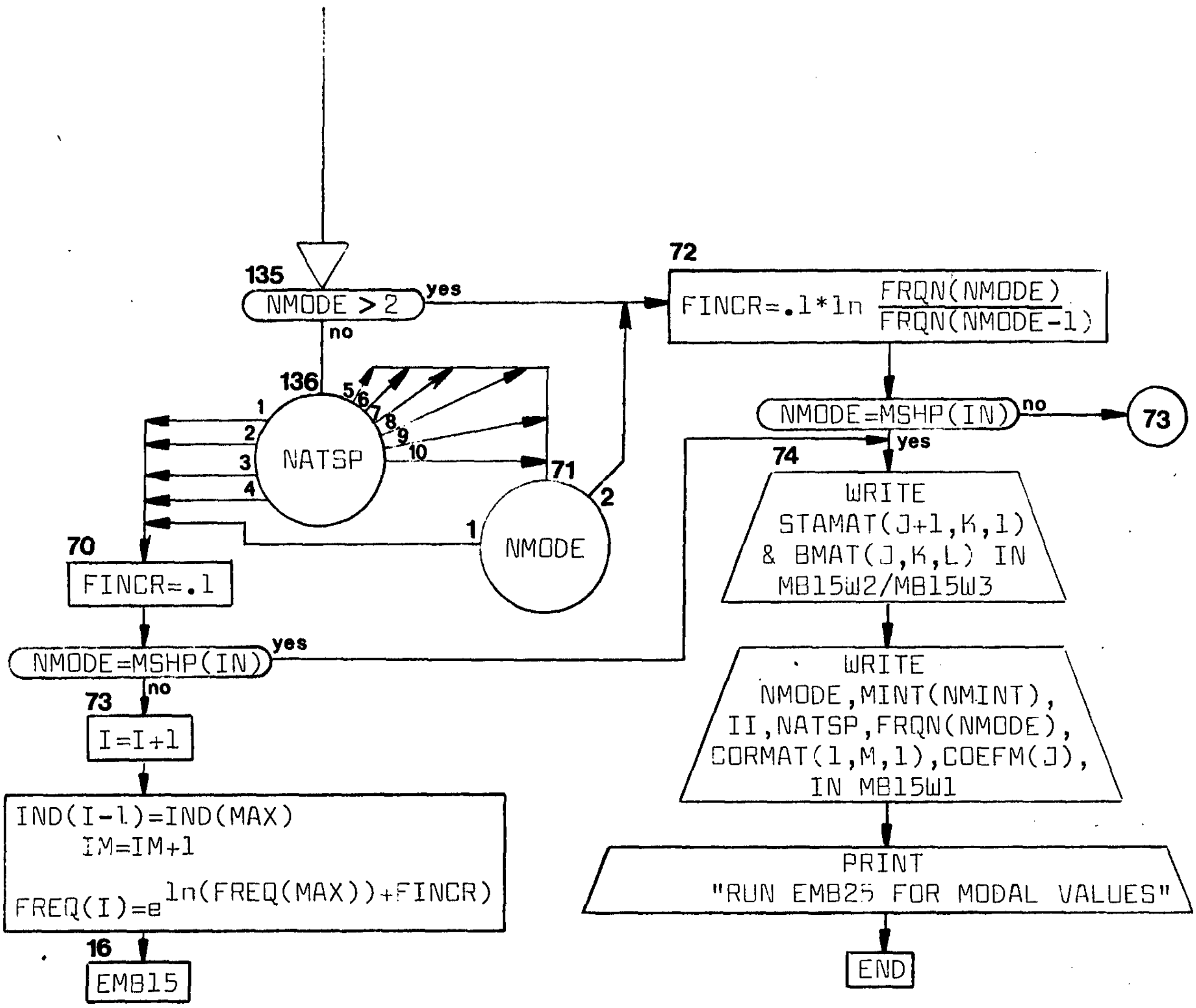
Saved Item EMB20

Figure 19

Flow Chart of EMB20

Figure 20




135
NMODE > 2
yes
no
72
FINCR=.1*ln FRQN(NMODE) / FRQN(NMODE-1)
136
NATSP
1
2
3
4
5
6
7
8
9
10
71
NMODE
1
2
NMODE=MSHP(IN)
no
73
yes
74
WRITE STAMAT(J+1,K,1) & BMAT(J,K,L) IN MB15W2/MB15W3
70
FINCR=.1
NMODE=MSHP(IN)
yes
no
73
I=I+1
WRITE NMODE,MINT(NMINT), II,NATSP,FRQN(NMODE), CORMAT(1,M,1),COEFM(J), IN MB15W1
IND(I-1)=IND(MAX)
IM=IM+1
FREQ(I)=e ln(FREQ(MAX))+FINCR)
PRINT "RUN EMB25 FOR MODAL VALUES"
16
EMB15
END

```
MB15W2     10:46   TUE.  09/23/69.

                                                                  STAMAT(1)
                                                                 .10000000E+01
                                                                -.17750559E+00
                                                                 .00000000E+01
                                                                 .00000000E+01

BMAT( 1)                                                          STAMAT( 2)
 .10000000E+01 .00000000E+01 .00000000E+01 .00000000E+01 .10000000E+01
  .00000000E+01 .10000000E+01 .00000000E+01 .00000000E+01 -.17750559E+00
  .00000000E+01 .00000000E+01 .10000000E+01 .00000000E+01  .00000000E+01
  .00000000E+01 .00000000E+01 .00000000E+01 .10000000E+01  .00000000E+01

BMAT( 2)                                                          STAMAT( 3)
 .10001575E+01 .34001071E+01 .42815261E-03 .48523679E-03 .39661947E+00
  .18526679E-03 .10001575E+01 .25185978E-03 .42815261E-03 -.17734827E+00
  .18239330E+00 .25011017E+01 .10001575E+01 .34001071E+01 -.26156623E+00
  .12981846E+01 .18239330E+00 .18526679E-03 .10001575E+01  .12658088E+01

BMAT( 3)                                                          STAMAT( 4)
 .10032907E+01 .17011188E+01 .10706052E-03 .60660020E-04 .96282933E-01
  .77433002E-02 .10032907E+01 .12600880E-03 .10706052E-03 -.17475818E+00
  .32597351E+02 .10453455E+03 .10032907E+01 .17011188E+01 -.37194143E+01
  .21714918E+03 .32597351E+02 .77433002E-02 .10032907E+01  .81612458E+02

BMAT( 4)                                                          STAMAT( 5)
 .10000000E+01 .00000000E+01 .00000000E+01 .00000000E+01 .96282933E-01
  .00000000E+01 .10000000E+01 .00000000E+01 .00000000E+01 -.17475818E+00
  .00000000E+01 .00000000E+01 .10000000E+01 .00000000E+01 -.37194143E+01
  .44544149E+03 .00000000E+01 .00000000E+01 .10000000E+01  .12450087E+03
```

Portion of Permanent File MB15W2

Figure 21

```
MB15W1     10:45    TUE.   09/23/69.

NMØDE= 4  MINT(NMINT)= 5  II= 1  NATSP= 2  FRQN=  .15672561E+03
CØRMAT(1) 0.0 1.0 0.0 0.0
CØEFM( 1)=   .72925182E-01
CØEFM( 2)=   .31183327E+00
CØEFM( 3)=   .31183327E+00
CØEFM( 4)=   .31229857E+00
CØEFM( 5)=   .95974937E-01
CØEFM( 6)=   .92346649E-01
CØEFM( 7)=   .92346649E-01
CØEFM( 8)=   .71525685E-01
CØEFM( 9)=   .64630752E-01
CØEFM(10)=   .18062243E+00
CØEFM(11)=   .18062243E+00
CØEFM(12)=   .24705218E+00
CØEFM(13)=   .11119480E+01
CØEFM(14)=   .82500390E+00
CØEFM(15)=   .14667209E+00
CØEFM(16)=   .14667209E+00
CØEFM(17)=   .10371283E+00
```

Permanent File MB15W1

Figure 22

```
RUN


EMB15      18:09   WED.  09/03/69.

IN EMB20
IN EMB21
IN  EMB20
SIZE AT LINE NØ. 105, 3009, 1810, 0

INPUT VALUES ØF II,IM,IN,NMØDE,FREQ(1),FINCR AND IND(1)
? ?1,2,2,2,5.,.2,1
FRQN( 1)=
? ?0.
DIFFERENCE VALUE( 2)=  .21752718E+01 FREQ=  .61070138E+01
DIFFERENCE VALUE( 3)=  .31549069E+01 FREQ=  .74591235E+01
DIFFERENCE VALUE( 4)=  .43915608E+01 FREQ=  .91105939E+01
DIFFERENCE VALUE( 5)=  .57982241E+01 FREQ=  .11127705E+02
DIFFERENCE VALUE( 6)=  .70811595E+01 FREQ=  .13591409E+02
DIFFERENCE VALUE( 7)=  .75630462E+01 FREQ=  .16600584E+02
DIFFERENCE VALUE( 8)=  .59732824E+01 FREQ=  .20275999E+02
DIFFERENCE VALUE( 9)=  .35757613E+00 FREQ=  .24765162E+02
DIFFERENCE VALUE(10)= -.11560836E+02 FREQ=  .30248236E+02

RØUGH FRQN( 2)=  .24929665E+02=   .39676701E+01HZ


ICRØSS=   9

DIFFERENCE VALUE(11)=  .75885236E-01 FREQ=  .24929665E+02
ICRØSS=  10

DIFFERENCE VALUE(12)=  .15826825E-01 FREQ=  .24964348E+02
DIFFERENCE VALUE(13)=  .32953923E-02 FREQ=  .24971572E+02
DIFFERENCE VALUE(14)=  .66740138E-03 FREQ=  .24973075E+02
DIFFERENCE VALUE(15)=  .17710266E-03 FREQ=  .24973380E+02
DIFFERENCE VALUE(16)= -.22733404E-04 FREQ=  .24973461E+02
ICRØSS=  15

DIFFERENCE VALUE(17)=  .47282049E-04 FREQ=  .24973452E+02
ICRØSS=  16

DIFFERENCE VALUE(18)=  .14763141E-04 FREQ=  .24973458E+02
DIFFERENCE VALUE(19)=  .30237404E-05 FREQ=  .24973459E+02
DIFFERENCE VALUE(20)=  .59870047E-06 FREQ=  .24973459E+02

FINAL FRQN( 2)=  .24973459E+02=   .39746402E+01HZ INTERP= 10

RUN EMB25 FØR MØDAL VALUES
```

Sample Output for Second Mode

Figure 23

(This page left intentionally blank)

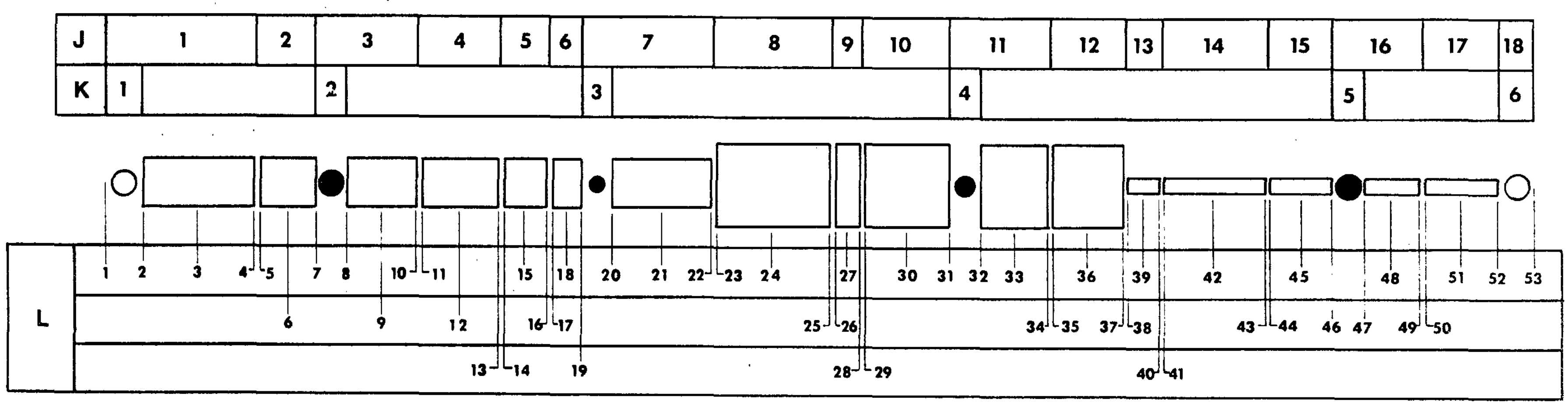
Computation of Eigenmodes and Stress Values

Lines 1-57 represent the input section of EMB25 (refer to Figures 25 and 26), which is based on the initial data set EMB10R, the output files MB10W2 and MB10W3 of EMB10 and the output files of EMB15/20. The program computes (VAL(I,L),I=1,6),L=1,LSUM+NSS+2) representing the four components of the state vectors plus the maximum bending and average shear stresses at each of the 53 values of distance DIST(L) from the left-hand support. The beam as partitioned by the point loads and property jump discontinuities is shown in Figure 24, which also shows the 53 stations corresponding to the L values. At each discontinuity, L is incremented to account for shear force change due to the point load as well as for stress changes due to the possibility of variations in cross-section properties PAR(5-7,J). The latter occur at L=22,23 and L=37,38; the remaining 10 discontinuities do not affect stress values as they correspond to changes in distributed load, material density or elastic modulus PAR(2-4,J). VAL(I,L) are first computed on the basis of existing state vectors, i.e. the STAMAT values as read in. The largest deflection (in absolute value) is then found (VALMAX) and compared with unity to form RATIO. The appropriate normalized component of Z_1, either W or V, is then multiplied by RATIO and a new series of state vectors obtained through the $USE EMB21 monitor line. These renormalized values are returned to statement 119 and a new sequence of VAL(I,L) obtained which will contain a maximum displacement of $\pm$ 1 inch.

Lines 59-69 establish the number and length of the subspan segments, XNOSEG(J) and SUBSEG(J) respectively; a maximum of 20 segments is assured (line 50), but this may be varied by the user at will. For the given beam no subspan > SSUM/20, hence XNOSEG(J) =2 for all J. Line 71 establishes the limit of L, LIM2, as the sum of the number of subspans NSS, the total number of segments LSUM, plus one station for each end of the beam. Lines 73-75 initialize VAL(I,L) using the state vector as read in or returned by EMB21. In line 81 J=IPAR(1,K) means that VAL(I,L) are to be computed for

the right side of a point load, and if NSS < IPAR(1,K) lines 85-87 compute the VAL(I,LIM2) and control passes to statement 103. If J≠IPAR(1,K) lines 93-97 compute the VAL(I,L) for the left side of a general subspan. Lines 99-147 are involved in the computation of VAL(I,L) for all the remaining stations within a subspan. SPAN is a summer of the segment lengths for each subspan and has a maximum of SS(J). The value of L at the beginning of a subspan is retained in LSAVE, to which the current SPAN is added to obatin the corresponding DIST(L) (line 103). As long as M< LIM1 control passes to statement 44 which considers the possibility of a massless field. The NO branch causes VAL(I,L) to be computed in accordance with Figure 3—the YES branch in accordance with Figure 2; control then passes to 42. When M=LIM1 the VAL(I,L) are computed for the right side of the subspan and J is incremented for the next subspan/point load.

In lines 149-165 renormalization is performed as described above, involving the compilation of the additional 1000 characters of EMB21, producing a total length of about 5400 (about 5300 machine words). If further modes of interest remain the YES branch of line 177 is taken and a notice printed to rerun EMB15. If NMODE=MINT(NMINT) =MSHP(NMSHP) the possibility of further support conditions is investigated in statement 102; the NO branch signifies problem completion. The YES branch causes a rerun EMB15 notice to be printed.



Assignment of Values to J, K, and L

Figure 24

```
$FILE EMB10R,MB10W2,MB10W3,MB15W1,MB15W2/MB15W3
DIMENSIØN NPAR(7),PAR(7,17),IPAR(1,20),SUBSEG(17),SS(17),
+ XNØSEG(17),CØEFM(17),BMAT(23,4,4),STAMAT(24,4,1),CØRMAT(24,4,1),
+ VAL(6,70),DIST(70),A(4),MINT(20)
READ(1,28) NSS,NØSUP,NMINT
28 FØRMAT(4X,3I5)
READ(1,29) (SS(I),I=1,NSS)
29 FØRMAT(4X,6F10.3)
READ(3,30) (NPAR(I),I=1,7)
30 FØRMAT(17X,7I3)
NPAR1=NPAR(1)
READ(3,31) (NPAR1,(IPAR(1,I),I=1,NPAR1)),SSUM
31 FØRMAT(//7X,*I4//33X,E15.8)
READ(2)
DØ 32 I=1,7
NPR=NPAR(I)
32 READ(2,33) (PAR(I,J),J=1,NPR)
33 FØRMAT(6X,4E15.8)
READ(4,16) NMØDE,MINT(NMINT),II,NATSP,(CØRMAT(1,M,1),M=1,4),
+ (CØEFM(J),J=1,NSS)
16 FØRMAT(6X,I2,14X,I2,5X,I2,8X,I2/9X,4F4.1/(11X,E15.8))
LIM=NSS+NPAR(1)
READ(5,18) (STAMAT(1,M,1),M=1,4); 18 FØRMAT(/(57X,E14.8))
DØ 19 J=1,LIM
READ(5,35) A(1),A(2); 35 FØRMAT(A6/A6)
DØ 19 K=1,4
READ(5,20) (BMAT(J,K,L),L=1,4),STAMAT(J+1,K,1)
20 FØRMAT(4E14.8,X,E14.8)
19 CØNTINUE
SEG=SSUM/20.; LSUM=ITER=0
DØ 1 J=1,NSS
IF(2-FIX(AINT(SS(J)/SEG))) 13,10,10
10 XNØSEG(J)=2; GØ TØ 17; 13 XNØSEG(J)=AINT(SS(J)/SEG)
17 LSUM=LSUM+FIX(XNØSEG(J))
1 SUBSEG(J)=SS(J)/XNØSEG(J)
LIM2=LSUM+NSS+2
119 DØ 50 K=1,4; VAL(K,1)=STAMAT(1,K,1); VAL(5,1)=VAL(3,1)*
+PAR(7,1)/PAR(5,1); 50 VAL(6,1)=VAL(4,1)/PAR(6,1)
K=1; L=2; J=0; DIST(1)=0.
12 J=J+1; DIST(L)=DIST(L-1)
IF(J-IPAR(1,K)) 3,4,3; 4 IF(NSS-IPAR(1,K)) 9,2,2
2 DØ 5 I=1,4; 5 VAL(I,L)=STAMAT(J+K,I,1)
VAL(5,L)=VAL(3,L)*PAR(7,J)/PAR(5,J)
VAL(6,L)=VAL(4,L)/PAR(6,J); K=K+1; GØ TØ 8
9 DØ 15 I=1,4; 15 VAL(I,L)=STAMAT(J+K,I,1)
VAL(5,L)=VAL(5,L-1); VAL(6,L)=VAL(4,L)/PAR(6,J-1); GØ TØ 103
3 DØ 14 I=1,4; 14 VAL(I,L)=STAMAT(J+K-1,I,1)
VAL(5,L)=VAL(3,L)*PAR(7,J)/PAR(5,J)
VAL(6,L)=VAL(4,L)/PAR(6,J)
8 SPAN=0.; LIM1=XNØSEG(J); LSAVE=L; M=0
EI=PAR(4,J)*PAR(5,J); 42 M=M+1; SPAN=SPAN+SUBSEG(J); L=L+1
DIST(L)=DIST(LSAVE)+SPAN
IF(LIM1-M) 44,40,44; 40 DØ 43 I=1,4; 43 VAL(I,L)=STAMAT(J+K,I,1)
VAL(5.L)=VAL(3.L)*PAR(7.J)/PAR(5.J); VAL(6.L)=VAL(4.L)/PAR(6.J)
```

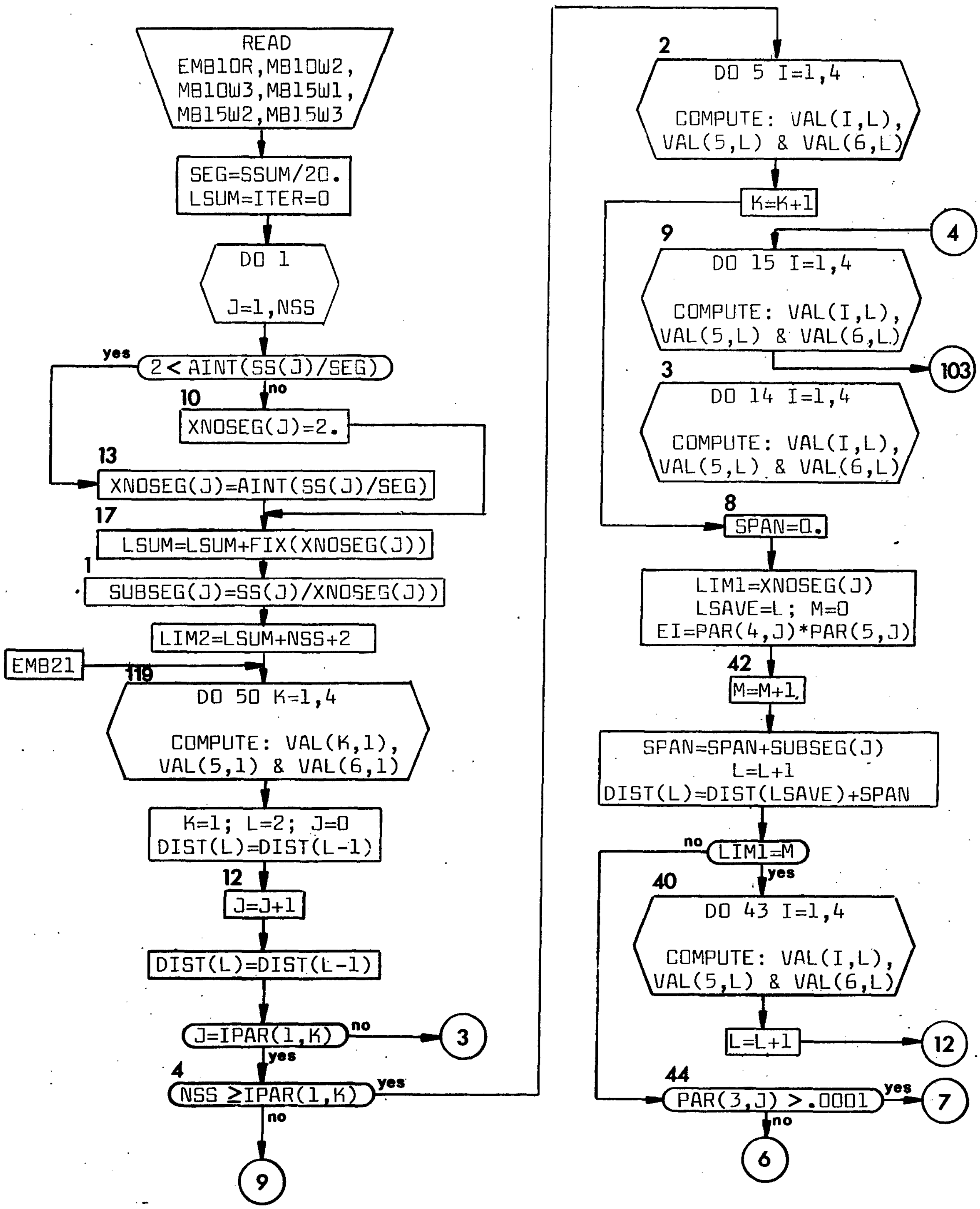
Program EMB25
Figure 25

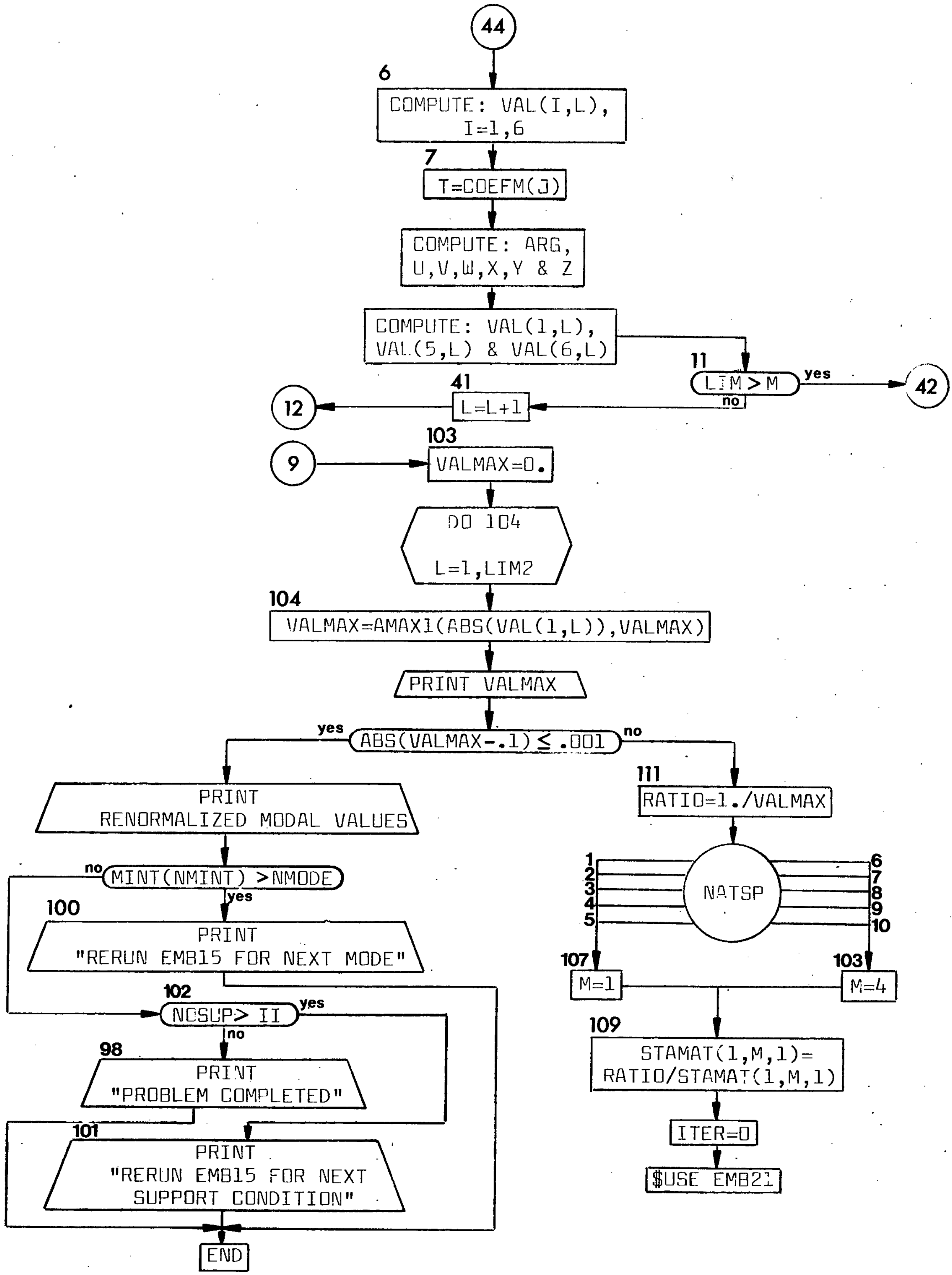
```
L=L+1; GØ TØ 12; 44 IF(PAR(3,J)-.0001) 6,6,7
6 VAL(1,L)=VAL(1,L-1)+VAL(2,L-1)*SPAN
+ +VAL(3,L-1)*SPAN**2/(2.*EI)+VAL(4,L-1)*SPAN**3/(6.*EI)
VAL(2,L)=VAL(2,L-1)+VAL(3,L-1)*SPAN/EI+VAL(4,L-1)*SPAN**2/(2.*EI)
VAL(3,L)=VAL(3,L-1)+VAL(4,L-1)*SPAN
VAL(4,L)=VAL(4,L-1); VAL(5,L)=VAL(3,L)*PAR(7,J)/PAR(5,J)
VAL(6,L)=VAL(4,L)/PAR(6,J); GØ TØ 11
7 T=CØEFM(J); ARG=T*SPAN
U=(EXP(ARG)+EXP(-ARG))/2.; V=CØS(ARG)
X=(EXP(ARG)-EXP(-ARG))/2.; Y=SIN(ARG)
W=U-V; Z=X-Y
VAL(1,L)=.5*(VAL(1,LSAVE)*(U+V)+VAL(2,LSAVE)*(X+Y)/T
+ +VAL(3,LSAVE)*W/(EI*T**2)+VAL(4,LSAVE)*Z/(EI*T**3))
VAL(5,L)=.5*(VAL(1,LSAVE)*W*EI*T**2+VAL(2,LSAVE)*Z*EI*T
+ +VAL(3,LSAVE)*(U+V)+VAL(4,LSAVE)*(X+Y)/T)*PAR(7,J)/PAR(5,J)
VAL(6,L)=.5*(VAL(1,LSAVE)*(X+Y)*EI*T**3+VAL(2,LSAVE)*W*EI*T**2
+ +VAL(3,LSAVE)*Z*T+VAL(4,LSAVE)*(U+V))/PAR(6,J)
11 IF(LIM1-M) 41,41,42
41 L=L+1
GØ TØ 12
103 VALMAX=0.; DØ 104 L=1,LIM2
104 VALMAX=AMAX1(ABS(VAL(1,L)),VALMAX)
PRINT 177,VALMAX; 177 FØRMAT(7HVALMAX=,E15.8)
IF(ABS(ABS(VALMAX)-1.)-.001) 110,110,111
111 RATIØ=1./VALMAX
GØ TØ(107,107,107,107,107,108,108,108,108,108), NATSP
107 M=1; GØ TØ 109; 108 M=4
109 STAMAT(1,M,1)=RATIØ*STAMAT(1,M,1); ITER=0
$USE EMB21
110 PRINT 178, "RENØRMALIZED MØDAL VALUES FØR NMØDE=",NMØDE
178 FØRMAT(I2/71HDIST ALØNG BEAM          DEFLECTIØN        BENDING
+ STRESS      SHEAR STRESS)
PRINT 179, (DIST(L),VAL(1,L),VAL(5,L),VAL(6,L),L=1,LIM2)
179 FØRMAT(E15.8,4X,E15.8,4X,E15.8,3X,E15.8)
IF(MINT(NMINT)-NMØDE)102,102,100
100 PRINT,"RERUN EMB15 FØR NEXT MØDE"
GØ TØ 99
102 IF(NØSUP-II) 98,98,101; 98 PRINT^"PRØBLEM CØMPLETED"; GØ TØ 99
101 PRINT^"RERUN EMB15 FØR NEXT SUPPØRT CØNDITIØN"
99 CØNTINUE
$ØPT SIZE
END
```

LENGTH
ABØUT 4400 CHARS.

Flow Chart of EMB25

Figure 26




44
6
COMPUTE: VAL(I,L), I=1,6
7
T=COEFM(J)
COMPUTE: ARG, U,V,W,X,Y & Z
COMPUTE: VAL(1,L), VAL(5,L) & VAL(6,L)
11
LIM > M
yes
no
42
41
L=L+1
12
103
9
VALMAX=0.
DO 104 L=1,LIM2
104
VALMAX=AMAX1(ABS(VAL(1,L)),VALMAX)
PRINT VALMAX
yes
ABS(VALMAX-.1) ≤ .001
no
PRINT RENORMALIZED MODAL VALUES
no
MINT(NMINT) >NMODE
yes
100
PRINT "RERUN EMB15 FOR NEXT MODE"
102
NCSUP> II
yes
no
98
PRINT "PROBLEM COMPLETED"
101
PRINT "RERUN EMB15 FOR NEXT SUPPORT CONDITION"
END
111
RATIO=1./VALMAX
NATSP
1
2
3
4
5
6
7
8
9
10
107
M=1
103
M=4
109
STAMAT(1,M,1)= RATIO/STAMAT(1,M,1)
ITER=0
$USE EMB21

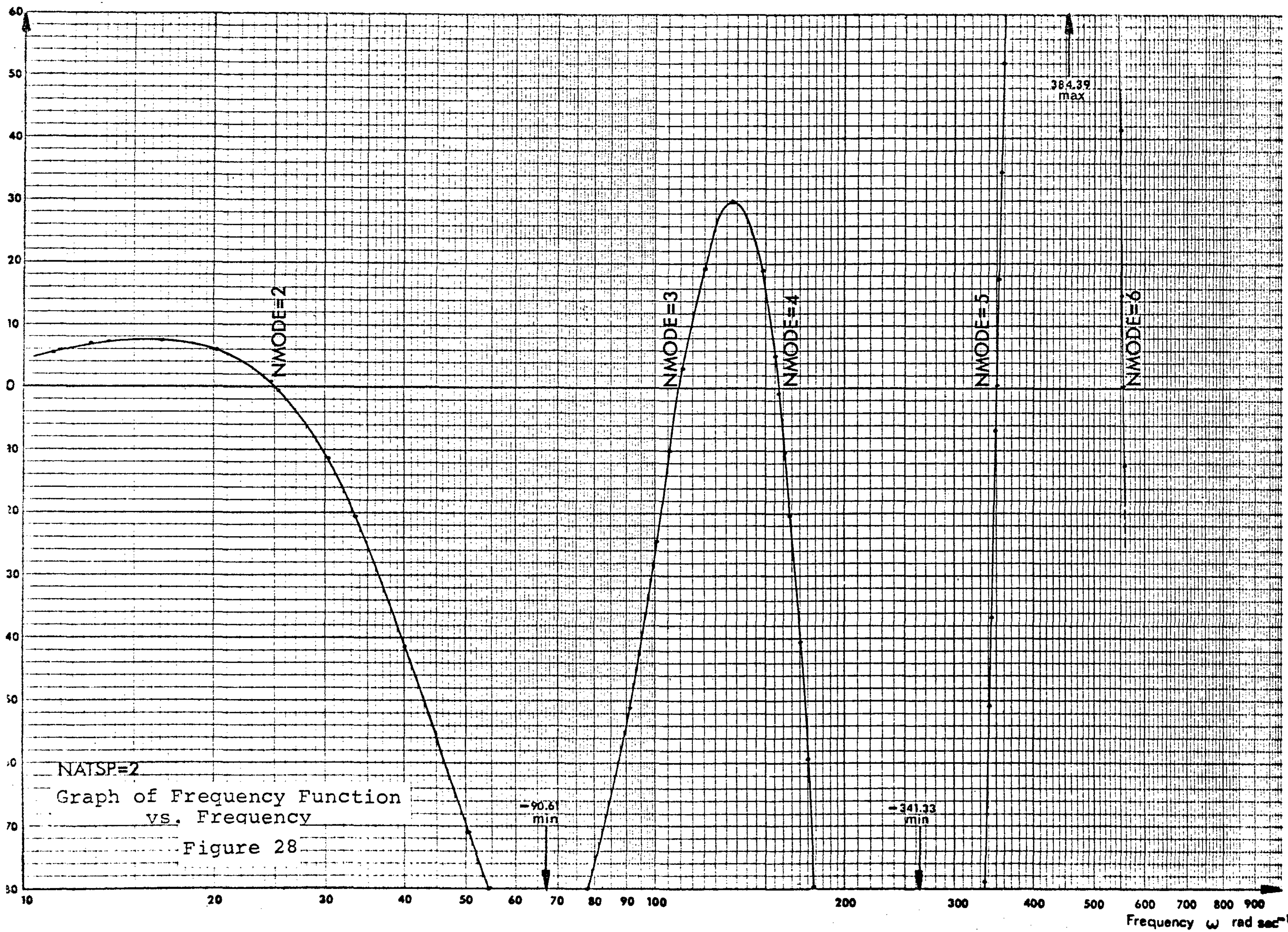
VALMAX= .58490432E+01
VALMAX= .10000119E+01
RENØRMALIZED MØDAL VALUES FØR NMØDE= 2

Eigenmode and Stress Values for Second Mode

Figure 27.

DIST ALØNG BEAM	DEFLECTIØN	BENDING STRESS	SHEAR STRESS
.00000000E+01	.17096813E+00	.00000000E+01	.00000000E+01
.00000000E+01	.17096813E+00	.00000000E+01	.00000000E+01
.17000000E+01	.13863570E+00	.74800825E+00	.42520182E-01
.34000000E+01	.10630383E+00	-.29724837E+00	.93335385E-01
.34000000E+01	.10630383E+00	-.29724837E+00	.93335385E-01
.42500000E+01	.90138766E-01	.40675727E+02	.46033042E+01
.51000000E+01	.73981174E-01	-.22423784E+01	.97509838E+01
.51000000E+01	.73981174E-01	-.22423784E+01	.23696611E+02
.61500000E+01	.54053567E-01	.53548799E+03	.27327400E+02
.72000000E+01	.34258465E-01	.97672862E+03	.31896800E+02
.72000000E+01	.34258465E-01	.97672862E+03	.31896800E+02
.84000000E+01	.11963786E-01	.17632168E+04	.33401524E+02
.96000000E+01	-.97661641E-02	.24719614E+04	.36110421E+02
.96000000E+01	-.97661641E-02	.24719614E+04	.36110421E+02
.10300000E+02	-.22083538E-01	.29774743E+04	.36105009E+02
.11000000E+02	-.34076703E-01	.34829734E+04	.36103775E+02
.11000000E+02	-.34076703E-01	.34829734E+04	.36103775E+02
.11450000E+02	-.41601325E-01	.38078715E+04	.36095519E+02
.11900000E+02	-.48979073E-01	.41328200E+04	.36088912E+02
.11900000E+02	-.48979073E-01	.41328200E+04	.32132053E+02
.13400000E+02	-.72358416E-01	.50961606E+04	.32087821E+02
.14900000E+02	-.93553749E-01	.60599847E+04	.32061441E+02
.14900000E+02	-.93553749E-01	.87235864E+03	.76947458E+01
.16600000E+02	-.11602906E+00	.10284320E+04	.76083735E+01
.18300000E+02	-.13816474E+00	.11855985E+04	.75397028E+01
.18300000E+02	-.13816474E+00	.11855985E+04	.75397028E+01
.18700000E+02	-.14331823E+00	.12217342E+04	.75215105E+01
.19100000E+02	-.14844937E+00	.12579511E+04	.75039793E+01
.19100000E+02	-.14844937E+00	.12579511E+04	.75039793E+01
.20450000E+02	-.16559389E+00	.13462522E+04	.33248485E+01
.21800000E+02	-.18245923E+00	.14828485E+04	-.44686274E+00
.21800000E+02	-.18245923E+00	.14828485E+04	-.51637540E+01
.22850000E+02	-.19537445E+00	.13934587E+04	-.90741855E+01
.23900000E+02	-.20811463E+00	.13434725E+04	-.12738841E+02
.23900000E+02	-.20811463E+00	.13434725E+04	-.12738841E+02
.24950000E+02	-.22048921E+00	.11553423E+04	-.17175322E+02
.26000000E+02	-.23235639E+00	.10107633E+04	-.21374701E+02
.26000000E+02	-.23235639E+00	.84257230E+05	-.35624502E+03
.26550000E+02	-.21418935E+00	.71825209E+05	-.39666836E+03
.27100000E+02	-.15263101E+00	.60441613E+05	-.42806465E+03
.27100000E+02	-.15263101E+00	.60441613E+05	-.42806465E+03
.28650000E+02	.10843820E+00	.18985358E+05	-.44156728E+03
.30200000E+02	.39777000E+00	-.78795382E+04	-.46541155E+03
.30200000E+02	.39777000E+00	-.78795382E+04	-.46541155E+03
.31150000E+02	.56774915E+00	-.34404013E+05	-.46526305E+03
.32100000E+02	.71891080E+00	-.60931984E+05	-.46516810E+03
.32100000E+02	.71891080E+00	-.60931984E+05	-.54045866E+00
.32950000E+02	.82807431E+00	-.60954241E+05	-.32738883E+00
.33800000E+02	.91054772E+00	-.60984404E+05	-.15306567E+00
.33800000E+02	.91054772E+00	-.60984404E+05	-.15306567E+00
.34900000E+02	.97764322E+00	-.60991760E+05	-.68837025E-01
.36000000E+02	.10000119E+01	-.61003762E+05	.39500943E-02
.36000000E+02	.10000119 E+01	-.61003762E+05	.39500943E-02

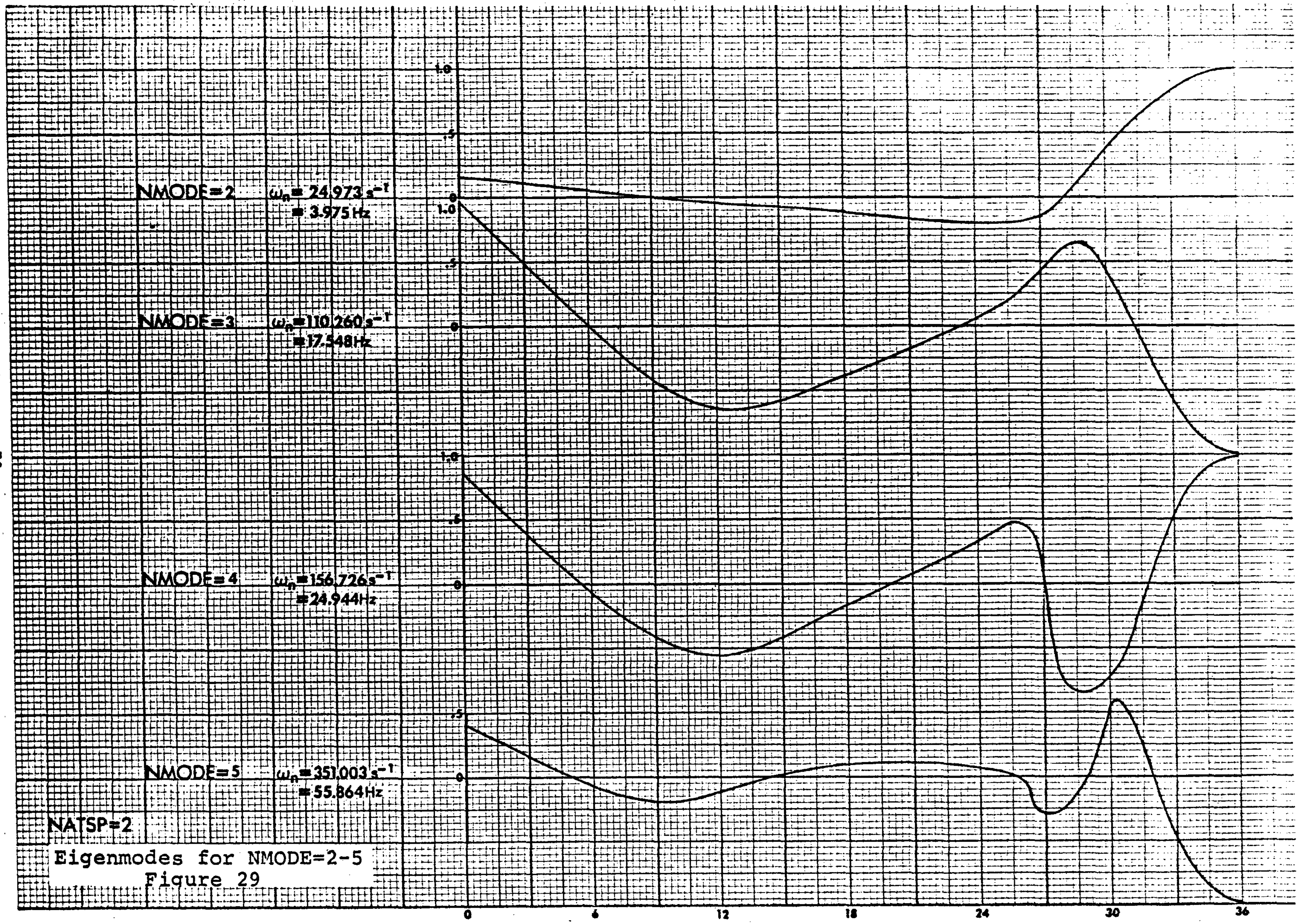
RERUN EMB15 FØR NEXT MØDE



NATSP=2

Graph of Frequency Function vs. Frequency

Figure 28



Eigenmodes for NMODE=2-5
Figure 29

BIBLIOGRAPHY

(1) Pestel, E., and F. Leckie: "Matrix Methods in Elastomechanics," McGraw-Hill Book Company, Inc., New York, pp. 204-213, 1963.

(2) Kuzanek, J.: "FAMSUB, a Computer Program for Determining the Frequencies and Mode Shapes of Uniform Beams," NASA Goddard Space Flight Center, Maryland, 1967.

(3) Salani, H., and H. Matlock: "A Finite-element Method for Transverse Vibrations of Beams and Plates," Texas Univ., Austin, 1967.

(4) Jeter, E.: "A Computer Program for Calculating the Natural Modes of a Non-uniform Beam," Naval Ordnance Test Station, China Lake, California, 1967.

(5) Tadahiko, K.: "On the Free Lateral Vibration of a Beam with Variable Cross Section," National Aeronautical Laboratory, Japan, 1962.

(6) Harris and Crede: "Shock and Vibration Handbook," Vol. I, Section 7, pp. 10-24, McGraw-Hill Book Company, Inc., New York, 1961.

DISTRIBUTION LIST

ADDRESSEE	NO. COPIES
Commanding General U.S. Army Tank-Automotive Command Warren, Michigan 48090 ATTN: Development & Engineering Directorate, AMSTA-R	1
Chief Scientist/Technical Director of Laboratories, AMSTA-CL	1
Chief Engineer, AMSTA-CR	1
Mobility Systems Laboratory, AMSTA-U	1
Physical Science Laboratory, AMSTA-Z	1
Vehicular Components and Materials Laboratory, AMSTA-B	1
Automotive Components Division, AMSTA-BA	4
Research Library Branch, AMSTA-BSL	3
Commanding General U.S. Army Materiel Command ATTN: AMCRD Washington, D.C. 20315	1
Commander Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
Office, Chief of Research and Development Department of the Army Washington, D.C.	1
Commanding General U.S. Army Mobility Equipment Research and Development Center Fort Belvoir, Virginia 22C60	1
Harry Diamond Laboratories ATTN: TECHNICAL REPORTS GROUP Washington, D.C.	1

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) TACOM Warren, Michigan	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

FAMSNUB, A computer program for determining the frequencies and mode shapes of non-uniform beams.

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final

5. AUTHOR(S) (Last name, first name, initial)

McFarland John L.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
Sep 1969	57	6

8a. CONTRACT OR GRANT NO. b. PROJECT NO. 1T0 61101A91A c. Task Area 00 d. Work Unit No. 205 EH	9a. ORIGINATOR'S REPORT NUMBER(S) 10561 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

Qualified users may obtain copies of this report from DDC.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Technical Director of Labs USATACOM

13. ABSTRACT FAMSNUB is a digital computer program, written in single precision FORTRAN IV for the Philco-Ford/GE 265 time-sharing system, to determine the eigenfrequencies and eigenmodes of vibration in pure bending of loaded, piece-wise continuous single-span brams supported in any one of the ten possible combinations of the following end conditions: free, guided, hinged and clamped. Continuously variable beam properties may be accommodated by appropriate subdivision of the span. The distributed mass of the beam itself is taken into account by means of field transfer matrices; this represents an advance over the usual lumped-parameter approach. To preserve numerical accuracy at the higher mode values, a modified transfer matrix method is employed. Verification solutions have been obtained on the IBM 360-50/65 using the double precision facility.

DD FORM 1 JAN 64 1473

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
COMPUTER PROGRAM						
FREQUENCIES AND MODE SHAPES						
NON-UNIFORM BEAMS						
EIGENFREQUENCIES						
EIGENMODES						
VIBRATION						
VARIABLE BEAM PROPERTIES						
MODIFIED TRANSFER MATRIX						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
____."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
____."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, *or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Idenfiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.